The Labor-Market Origins of Cyclical Skewness*

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Abstract

I use Danish administrative data 1980-2013 to study the underlying mechanisms generating fluctuations in income-shock moments. I partition the population into 37 narrowly defined educational categories and document the cyclicality of labor-income shock moments for each category separately. For the individual educational categories, mean income growth is strongly correlated with income-growth skewness, with an average correlation of $0.87 - 0.88$. The connection between income-growth skewness and mean income growth is not only strong in the time dimension, but also in the cross section. Across the 37 educational categories, the correlation between mean income growth and income growth skewness is $0.93 - 0.96$. Labor-market frictions together with variations in productivity growth can generate the relationship between mean income growth and income growth skewness. In a quantitative job-ladder model, variations in productivity growth quantitatively capture both the time-series and cross-sectional relationship.

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1 Introduction

Cyclical movements in income risk are commonly argued to be an important contributor to the cyclicality of asset prices (Constantinides and Duffie (1996), Storesletten et al. (2007), Schmidt (2016)), the welfare costs of business cycles (Storesletten et al. (2001), Krebs (2003, 2007), De Santis (2007)) and the cyclicality of consumption expenditures (Challe et al. (2017), McKay (2017), Ravn and Sterk (2017)). In this paper, I explore the underlying labor-market processes generating the cyclicality of measured income risk.

First, I use Danish administrative data 1980-2013 to document the cyclical variations in income risk. In a recent paper, Guvenen et al. (2014) showed that US recessions are associated with more individuals experiencing large decreases in labor income and fewer individuals experiencing large increases in labor income, while the number of individuals experiencing small changes to their labor income is comparatively stable. In statistical terms, this pattern can be summarized by saying that the skewness of the labor income growth distribution is cyclical while the dispersion is acyclical. In the Danish administrative data, mean labor income growth is strongly correlated with labor income growth skewness, with a correlation of $0.88-0.90$ for one-year income growth, three-year income growth, and five-year income growth. In contrast, the dispersion of the income growth distribution is basically uncorrelated with mean income growth. This reaffirms the findings of Guvenen et al. (2014) in a Danish context.

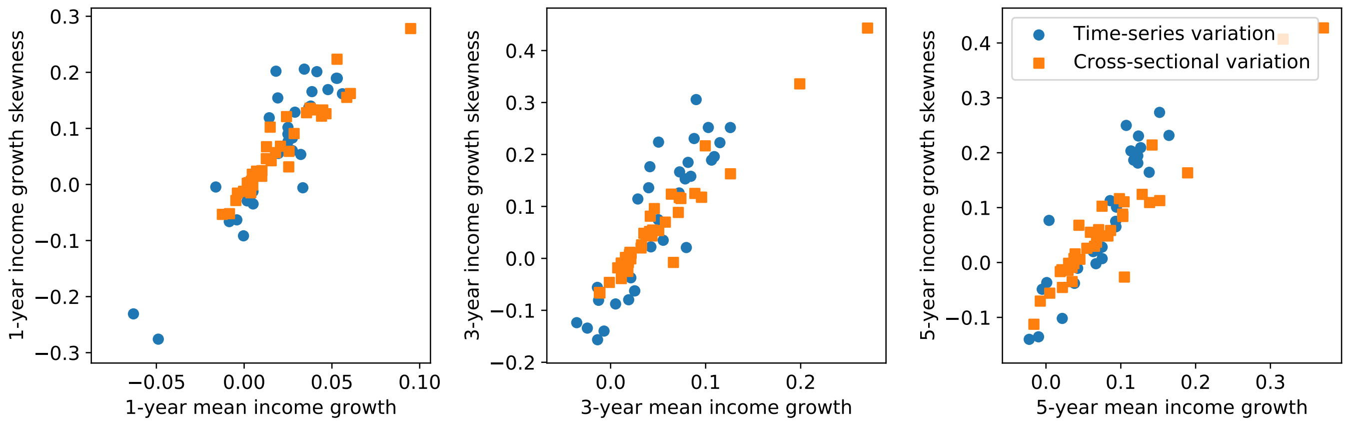
The macro-level relationship could potentially be a composition effect, with no corresponding relationship for the different labor markets comprising the economy. To explore cyclical income risk for different labor markets, I partition the population into 37 narrowly defined educational categories and show that almost all categories exhibit a high correlation of mean income growth and income growth skewness. [1] Mean income growth at the educational-category level is strongly correlated with income growth skewness, with an average correlation of $0.87-0.88$ for the different horizons. As for the full population distribution, the dispersion of the income growth distribution is uncorrelated with mean income growth.

Furthermore, the connection between income growth skewness and mean income growth is not only strong in the time dimension, but also in the cross section. Educational categories with high mean income growth 1980-2013 are the educational categories with most positive skewness of their income growth distribution. Across the 37 educational categories, the correlation between mean income growth and income growth skewness is $0.93-0.96$.

The empirical findings are summarized in Figure 1.1. In the figure, the blue circles show the year-specific mean income growth and income growth skewness for the full population 1980-2013. The orange squares show the educational-category specific mean income growth and income growth skewness (pooling all years).

[1] Educational categories overlap with both industry and occupation categories (for example, a medical doctor generally works with medicine in the health-care sector), but have the advantage that an individual's education is largely pre-determined when he or she enters the labor market. In contrast, with industry and occupation partitions, selective industry or occupation switching introduces bias in the estimates of income risk.

Figure 1.1: The mean income growth and the income growth skewness for all years (1980-2013, blue circles) and for all educational categories (37 educational categories, orange squares).



As is evident from Figure 1.1, the time-series variation and the cross-sectional variation line up closely. It is natural to conjecture that the cross-sectional and time-series relationships share a common root.

I provide a mechanism generating the mean-skewness relationship, built on two key assumptions: There is variation in productivity growth over time and across educational categories and there are labor market frictions that make wages not respond one-to-one in the short run to productivity growth. If individual wages are updated only occasionally in an environment with productivity growth, then when they are updated the wage jumps to catch up with productivity growth. In an environment with higher productivity growth, the income jumps are comparatively larger, generating both positive mean income growth and more positive skewness of the income growth distribution. In a theoretical reduced-form setting, I show that such a framework gives first-order movements in skewness but only second-order movements in dispersion in response to variations in the growth rate. To evaluate the mechanism quantitatively, I introduce growth into a job-ladder model. The parameter values are taken directly from Bagger et al. (2014), who estimate a quantitative job-ladder model with sequential auctioning on Danish matched employer-employee data. Through the lens of the job-ladder model, variations in the job-finding rate, the job-separation and the offer-arrival rate for employed workers cannot generate the variation in mean income growth and income growth skewness that we documented. However, in the model framework, productivity growth quantitatively generates the relationship of mean income growth and income growth skewness at the three-year and five-year horizons, both in the cross section and in the time-series dimension. In the cross section, modelling different educational categories as identical except for their steady state productivity growth rates, productivity growth differences quantitatively account for the cross-sectional relationship of mean income growth and income growth skewness. In the time-series dimension, shocks to the productivity growth of an educational category generate a high correlation between mean income growth and income growth skewness, as well as quantitatively close relative magnitudes of the variations in the mean income growth and income growth skewness.

In the quantitative job-ladder model, wages are determined by piece-rate contracts. Therefore, for wages not to respond one-to-one with productivity growth we assume that growth is only reflected in the productivity of potential future matches, not the productivity of ongoing matches. In other words, productivity is embodied for the particular match. The productivity growth mechanism is consistent with other well-established facts. Viewing the growth of the offer-productivity distribution as an important source of the labor-market business cycle implies that the job-to-job transition rate is strongly pro-cyclical. The pro-cyclicality of the job-to-job transition rate is well documented and a recent series of papers have particularly emphasized the connection between the job-to-job transition rate and the mean labor income growth.[2] In contrast, job-separation rate shocks generate a counterfactual negative comovement of mean income growth and the job-to-job transition rate.

1.1 Related Literature

The effects of fluctuations in income risk on welfare, consumption and savings have been studied extensively in the literature. Storesletten et al. (2001), Krebs (2003, 2007) and De Santis (2007) study the effects of cyclical income risk on the welfare costs of business cycles. Constantinides and Duffie (1996), Storesletten et al. (2007) and Schmidt (2016) study the asset pricing implications of cyclical income risk. Challe et al. (2017), McKay (2017), and Ravn and Sterk (2017) study the effects on consumption dynamics.

On the empirical side, Storesletten et al. (2004) argued using PSID data that US recessions were associated with higher variance of permanent shocks to household income after taxes and transfers. Our empirical work directly builds on a recent literature, starting with Guvenen et al. (2014), which argues that the fluctuations in individual labor income risk are shifts in the skewness of the income growth distribution. Guvenen et al. (2014) study, using US social security administrative data, the cyclicality of the skewness and dispersion of the labor income growth distribution and find that the dispersion is essentially acyclical while the skewness is more negative in recessions. Busch et al. (2016) do a similar analysis for Germany, Sweden and the US using the PSID and broadly find the same cyclical skewness in all three countries. Blass-Hoffmann and Malacrino (2016) also aim to understand the underlying labor market mechanisms that generate cyclical skewness of labor income. They use Italian administrative data and do a decomposition analysis where they decompose the cyclicality of the skewness into contributions from wages and contributions from employment time. They find that the cyclicality of skewness is driven by cyclical changes in employment time, which taken at face value contradicts our result that cyclical skewness is driven by productivity growth shocks. Importantly,

[2] See Faberman and Justiniano (2015), Moscarini and Postel-Vinay (2016), Moscarini and Postel-Vinay (2017) and Karahan et al. (2017).

they restrict their analysis to one-year income growth, which is exactly the horizon at which variations in productivity growth do not provide a quantitative account for the fluctuations in income growth skewness. A potential reconciliation of our respective findings is that employment is important for the one-year income growth skewness, while job-to-job transitions generated by productivity growth are more important for the longer horizons.

The results of this paper are interpreted with a job-ladder model (Burdett and Mortensen (1998), Bontemps et al. (2000), Postel-Vinay and Robin (2002)). The paper shares the ambition to understand the statistical moments of income dynamics through a job-ladder model with Postel-Vinay and Turon (2010) and Hubmer (2016). Postel-Vinay and Turon (2010) estimate a standard ARMA process for labor income on a job-ladder model calibrated to match labor market transitions, and find that the estimated ARMA parameters are close to directly estimated parameters from the British Household Panel Survey. Hubmer (2016) studies a job-ladder model with risk aversion, wealth accumulation, and endogenous search effort, and finds that it generates large negative skewness and high excess kurtosis of the income growth distribution, as documented by Guvenen et al. (2016) for the US using social security data. This paper contributes to the project of understanding income moments through job-ladder models by studying the cyclicality of measured income risk rather than steady state moments, as well as the cross-sectional correlation for income risk and income growth.

1.2 Outline of the Rest of the Paper

In Section 2, we describe the data and the empirical methodology. In Section 3, we analyze the relationship between mean income growth and income growth skewness for the full population, for each educational category separately, and in the cross section. Section 4 describes the model and the model experiments. Section 5 concludes.

2 Data and Empirical Method

This paper studies the full male Danish population age 25-59 from 1980 to 2013. To be in the sample, the person has to be registered as living in Denmark by December 31st of the year.

The object of study is the labor income distribution for different educational categories. In this section, I introduce the measures of income, describe the educational categories, and describe the sample selection.

Income The income data are collected from the Danish tax authorities. The labor income measure includes pre-tax salaried income (including benefits and the value of stock options) and net profit from self-employment (excluding capital income and expenses), but excludes employer contributions to retirement funds. All earnings are normalized by consumer price index with 2015 as base year.

Table 2.1: Number of observations (1980-2013) under the sample selection criteria.

Sample restrictions	Year-person observations
Danish male 25-59	43,555,230
+ Income > 12 000 DKK	38,180,100
+ Educ. Req.	30,357,199

Education The education data are from the Danish student registry and the Danish qualifications registry. For each individual, his highest degree as of October 1st is reported. The degrees are reported very narrowly and some of the reported degrees are difficult to interpret.[3] To facilitate interpretation and quantitative analysis, I aggregate the degrees by the International Standard Classification of Education (ISCED 2013) into coarser yet fine categories based on detailed field and level of degree.

The educational level of some degrees, for example nursing, have changed over the sample period. When aggregating into ISCED 2013 categories I use the current classification of degrees to aggregate.

Up until 2006, migrants to Denmark were surveyed about their schooling and their response is used in conjunction with information from the Danish government. Since 2006, migrants have not been asked about their schooling and therefore recent migrants will not be in the sample when conditioning on education.

Income Growth The analysis is concerned with the growth of income over one, three and five years. k-year income growth in year t is computed as $\Delta_k \log y_t = \log y_{t+k} - \log y_t$. For example, the three-year income growth of 2008 refers to the growth between the years 2008 and 2011.

Sample Restrictions In order to focus on a population where almost everyone is either looking for employment or employed, I restrict attention to prime-age men, aged 25-59. When studying k-year income growth at time t , the individual has to be in the age span both in t and $t + k$ (so the oldest person in the sample when studying five-year income growth is 55).

Observations with income less than 12,000 DKK (2015 prices, approximately 1,900 US dollars) are dropped. When conditioning on education, I restrict attention to educational categories which have had at least 2500 individuals in the sample for all years between 1980 and 2013. This leaves us with 37 educational groups which span approximately 80 percent of working-age men.

Table 2.1 summarizes the sample selection. A list of the 37 educational categories is shown in Table A.1 in Appendix A.

Empirical Method For each year and each individual, I compute the log income growth at the one-year, three-year, and five-year horizons. Compiling the list of all the (log) income growth rates at a given horizon

[3] For example, 2490 corresponds to "agrarian economist" and 2491 to "green diploma module 4".

for a given year gives a distribution of income growth, for example the distribution of three-year income growth from 1987 to 1990. I then study how the dispersion and skewness of the income growth distribution vary over time and its comovement with the mean income growth.

As measures of the dispersion of the income growth distribution, I employ the difference between the 90th percentile and the 10th percentile of the distribution, $\mathsf{P90} - \mathsf{P10}$ dispersion, and the standard deviation of the distribution.

The skewness measures aim to capture the degree to which the distribution is asymmetric. In studying theoretical distributions, the standard measure of skewness is the normalized third moment of the distribution, defined by the following expression for a random variable X:

$$m_3 = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right]$$

where $\mu = E[X]$ is the expected value of X and $\sigma = \sqrt{E[(X-\mu)^2]}$ is the standard deviation of X.

However, the normalized third moment gives a lot of weight to outliers. The baseline measure of this paper, Kelley's skewness, is defined as

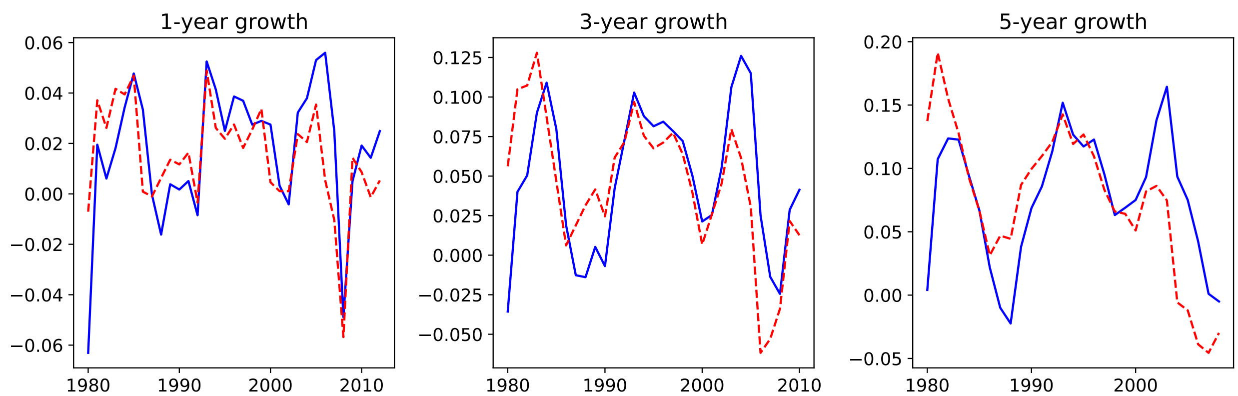
$$K = \frac{(P90 - P50) - (P50 - P10)}{P90 - P10},$$

where P90, P50, and P10 are the 90th percentile, the median and the 10th percentile of the distribution. Kelley's skewness is a more robust measure than statistical skewness, since it is insensitive to the outliers outside of the $P10 - P90$ range. Another advantage is that Kelley's skewness has a clear interpretation. It measures how much of the total dispersion, $P90 - P10$, stems from the right tail, $P90 - P50$, compared to the left tail, $P50 - P10$. If Kelley's skewness is 1, then all of the $P90 - P10$ dispersion is located in the right tail, while if it is -1 then the $P90 - P10$ dispersion resides in the left tail.

As the measure of the business cycle, I use the mean labor income growth rather than e.g., GDP per capita growth. Mean labor income growth is a more direct measure of the business cycle relevant for the labor market than GDP per capita growth. Also, using mean labor income growth as our business cycle measure helps us later when we study the business cycle for the different educational categories since the mean labor income growth for an educational category is readily observable while the educational-category specific GDP growth is not.

GDP per capita growth and the mean labor income growth are closely correlated, as shown in Figure 2.1, with one qualification: The mean labor income growth at the three-year and five-year horizons move with a lag of one year compared to real GDP per capita growth. The correlation of the two measures is 0.67 at the one-year horizon, 0.75 at the three-year horizon, and 0.73 at the five-year horizon (with a one-year lag in mean labor income growth for the longer horizons).

Figure 2.1: *Comovement of mean labor income growth (blue, solid) and real GDP per capita growth (red, dashed) at the one-year, three-year, and five-year horizons.*



3 Empirical Results on Income Growth Risk and Mean Income Growth

3.1 Cyclical Income Growth Risk for the Total Population

In the left of Figure 3.1, I plot the Kelley's skewness of the income growth distribution together with the mean of the income growth distribution, at the one-year, three-year and five-year horizons. Appendix A shows the corresponding graphs for the statistical skewness and standard deviation in Figure A.1.

The relationship between the skewness and mean is close, with a correlation of $0.87 - 0.89$. This contrasts with the right part of Figure 3.1, where we plot the P90-P10 dispersion of the income growth distribution together with the mean of the distribution. As is clear from the figure, the P90-P10 dispersion of the income growth distribution does not comove with the mean income growth. The correlation ranges from -0.25 to 0.21 at the different horizons. At the ten percent level we cannot reject the null hypothesis of no relationship between the dispersion measures and mean income growth in a regression framework. The regression results are shown in Appendix A, Table A.2.

Table 3.1, displays the correlations at all horizons for both the robust measures, Kelley's skewness and P90-P10 dispersion, as well as the standard measures, statistical skewness and standard deviation. The two skewness measures are highly correlated with mean labor income growth with a correlation of $0.87 - 0.90$. The two dispersion measures are slightly negatively correlated with the mean growth rate at the one-year horizon, essentially uncorrelated at the three-year horizon, and slightly positively correlated at the five-year horizon. In conclusion, the two measures of skewness move closely with the mean labor income growth, while the two measures of dispersion do not.

Figure 3.1: *Kelley's skewness of the labor income growth (blue solid, left) comoves with the mean of labor income growth (red dashed). The P90-P10 dispersion (blue solid, right) does not.*

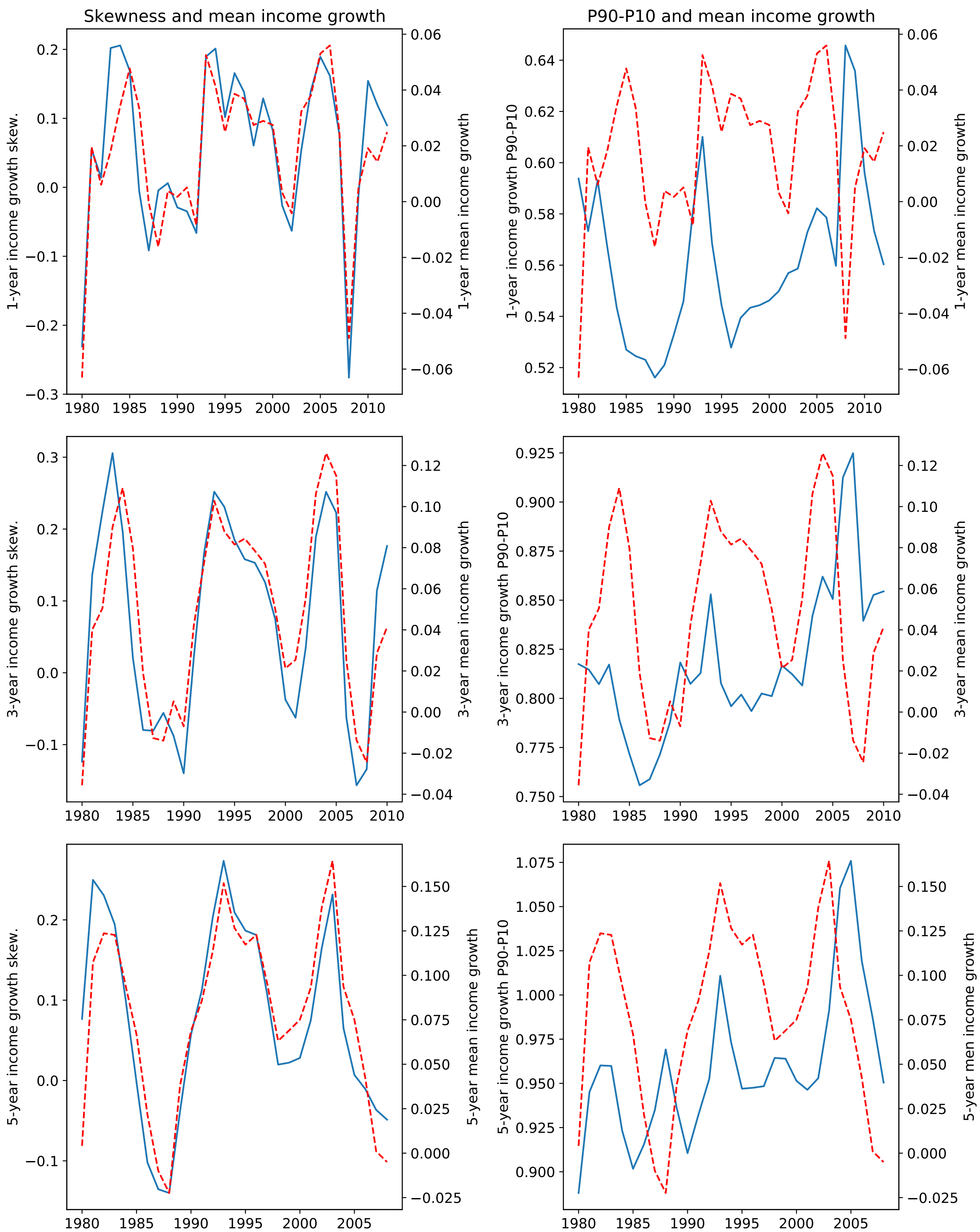


Table 3.1: *Correlation between the mean labor income growth and the skewness, Kelley's skewness, standard deviation and P90-P10 dispersion over the period 1980-2013, at the one-year, three-year, and five-year horizons.*

	1-year growth	3-year growth	5-year growth
Kelley's skewness	0.89	0.88	0.90
Skewness	0.88	0.88	0.87
P90-P10 dispersion	-0.25	0.02	0.21
Standard deviation	-0.26	0.03	0.15

In Section 4, I provide a labor-market mechanism generating this connection between mean income growth and income growth skewness. Before that, I document the cyclicality of income risk for the different educational categories.

3.2 Cyclical Income Growth Risk for the Educational Categories

Next we turn our focus to the relationship of income growth skewness and income growth dispersion with the mean income growth for the different 37 educational categories.

In Table 3.2, the average correlations of skewness, Kelley's skewness, the standard deviation and the P90-P10 dispersion with mean labor income growth at the different horizons are shown.

The correlation between the labor income growth skewness and the mean labor income growth is very high for almost all educational categories, with a few exceptions discussed below. The population-weighted average correlation is $0.87-0.88$ for Kelley's skewness and $0.76-0.78$ for statistical skewness. In Figure 3.2, we show the evolution of Kelley's skewness and mean labor income growth for the educational category with the correlation closest to the mean, *Engineering and engineering trade, Motor vehicles, ships and aircrafts – Vocational upper secondary education (completed, without direct access to tertiary education).*

In contrast, the average correlation with mean growth for the standard deviation and the P90-P10 dispersion are close to zero at the educational-category level for all horizons. The mean correlation is slightly negative, ranging from -0.05 to -0.17, but much weaker than for skewness and Kelley's skewness.

Although most educational categories have a high correlation between mean labor income growth and the labor income growth skewness, there are four educational categories with a correlation below 0.6 at some horizon. At the one-year horizon, two educational categories have a correlation below 0.6, teachers and police.[4] At the three-year and five-year horizons, in addition to the two aforementioned categories, medical doctors and child-and-youth-care professionals also have a correlation below 0.6.[5] We conjecture that wage

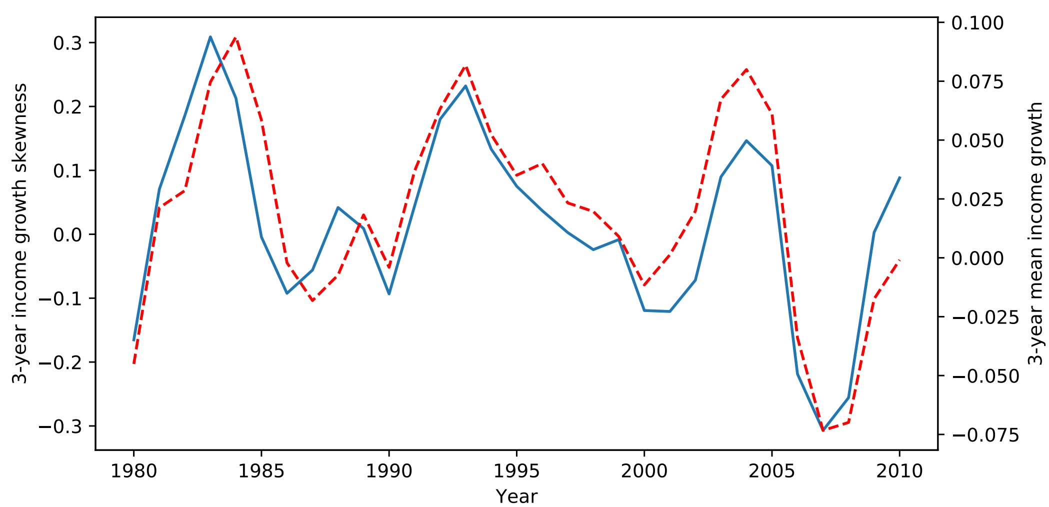
[4] *Education, Teacher training without subject specialization (Professional Bachelor's, First degree, 3-4 years)* and *Security services, Protection of persons and property (Professional Bachelor's, First degree, 3-4 years).*

[5] *Health, Medicine (Academic Master's degree following a Bachelor's degree)* and *Welfare, Child care and youth services (Professional Bachelor's, First degree, 3-4 years).*

Table 3.2: *Summary statistics, for all 37 educational categories, of the correlation of mean income growth with income growth skewness, Kelley's skewness, standard deviation and P90-P10. The mean and standard deviations are computed weighing the educational categories by population size.*

	Mean correlation	Standard deviation	Min. correlation	Max. correlation
1-year labor income growth K's skew.	0.87	0.14	0.22	0.97
3-year labor income growth K's skew.	0.88	0.14	0.18	0.97
5-year labor income growth K's skew.	0.88	0.15	0.07	0.96
1-year labor income growth skew.	0.78	0.20	-0.08	0.95
3-year labor income growth skew.	0.78	0.24	-0.34	0.93
5-year labor income growth skew.	0.76	0.21	-0.06	0.93
1-year labor income growth P90-P10	-0.17	0.33	-0.47	0.93
3-year labor income growth P90-P10	-0.12	0.36	-0.55	0.89
5-year labor income growth P90-P10	-0.05	0.39	-0.64	0.87
1-year labor income growth s.d.	-0.19	0.28	-0.45	0.80
3-year labor income growth s.d.	-0.11	0.31	-0.48	0.78
5-year labor income growth s.d.	-0.05	0.35	-0.59	0.84

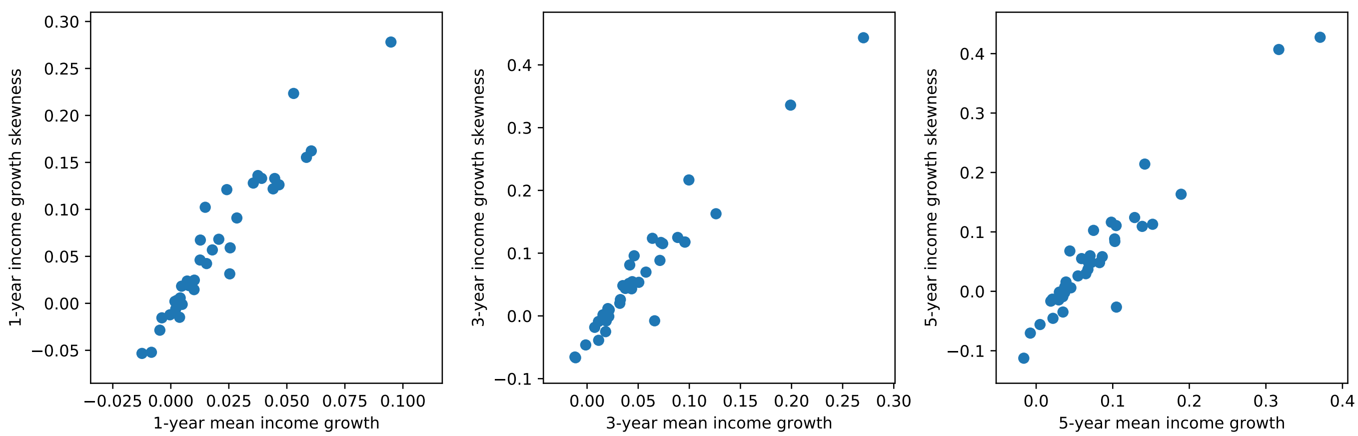
Figure 3.2: *Example of an average educational category, 'Engineering and engineering trade, Motor vehicles, ships and aircrafts – Vocational upper secondary education (completed, without direct access to tertiary education)'. The correlation of three-year labor income growth Kelley's skewness and three-year mean labor income growth is* 0.88*, equal to the (population weighted) average.*



determination for teachers, police, medical doctors and child-and-youth-care professionals is centralized. A centrally bargained wages can severe the link between skewness and the mean labor income growth in the following way: If wages increases are negotiated nationally, a centrally bargained wage will increase wages proportionally across the board, leaving the skewness of the distribution unaffected.

3.3 Income Growth Risk in the Cross Section

Figure 3.3: *Scatter plot of income growth Kelley's skewness and mean income growth in the cross section (37 educational categories), at the one-year, three-year and five-year horizons.*



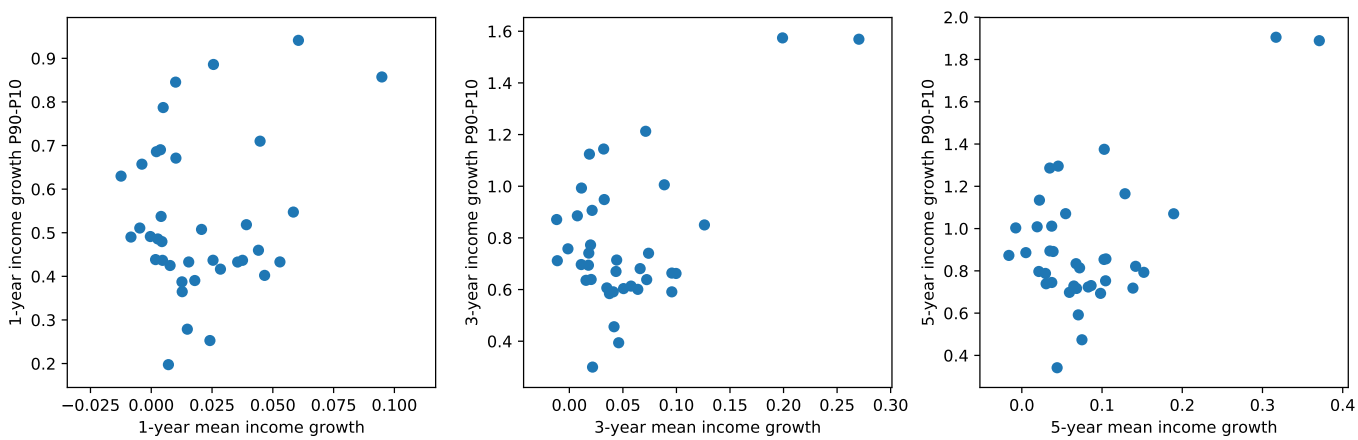
Next, we turn to the cross-sectional correlation between the mean labor income growth and the labor income growth distribution skewness. Ss before, the log labor income growth is computed at the one-year, three-year, and five-year horizons for each year, but now all the years are pooled together when constructing the income growth distribution.

Table 3.3 shows the cross-sectional correlations of mean labor income growth with Kelley's skewness, skewness, P90-P10 dispersion, and standard deviation at the one-year, three-year, and five-year horizons. Kelley's skewness has a correlation of $0.96 - 0.97$ for the different horizons, an arguably remarkably high correlation. Figure 3.3 shows scatter plots of the one-year, three-year and five-year mean income growth with income growth Kelley's skewness. Statistical skewness has a relatively high correlation, 0.68, at the short horizon, but the statistical skewness has a lower correlation at the longer horizons (0.56 for the three-year horizon and 0.50 for the five-year horizon). The two measures of dispersion, P90-P10 dispersion and the standard deviation, show a small correlation at the one-year horizon but a correlation at the five-year horizon $(0.50 - 0.59)$. Figure 3.4 shows a scatter plot of the mean income growth and the income growth P90-P10 dispersion at the one-year, three-year, and five-year horizons.

Table 3.3: *Correlation with mean labor income growth in the cross section for the 37 educational categories.*

	1-year growth	3-year growth	5-year growth
Kelley's skewness	0.96	0.97	0.96
Skewness	0.68	0.56	0.50
P90-P10 dispersion	0.21	0.53	0.59
Standard deviation	0.14	0.45	0.50

Figure 3.4: *Scatter plot of income growth P90-P10 dispersion and mean income growth in the cross section (37 educational categories), at the one-year, three-year and five-year horizons.*



3.4 Summary of Empirical Results

The correlation between mean income growth and income growth skewness is very high for both the full population and the educational categories in the time-series dimension, as well as in the cross section across educational categories. Measured cyclical income risk, as captured by cyclical skewness of the income growth distribution, is the time-series aspect of a tight link between mean income growth and income growth skewness which is also present in the cross section.

In the next section, I describe a mechanism through which productivity growth parsimoniously accounts for both the cross-sectional link between mean income growth and skewness as well as the time-series link between the two. I also show that conventional labor market characteristics fail to account for the link between mean income growth and skewness.

4 Model

In this section, I show that labor-market frictions together with productivity growth account for the relationship between mean income growth and income growth skewness. In the model framework, variations in the job-finding rate, the job-separation rate and the on-the-job offer arrival rate cannot account for the mean-skewness relationship.

The match between the model with productivity growth and data is quantitatively close. In the cross section, varying productivity growth captures both the level of skewness and the slope of the mean-skewness relationship. In the time-series dimension, shocks to productivity growth not only generate a high correlation between the skewness and mean, but also the relative magnitudes of the variations in the two.

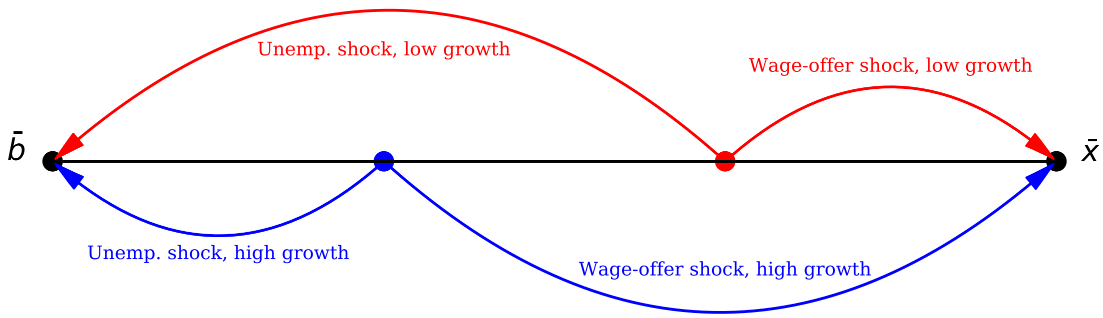
4.1 Lessons from a Simple Example

Before introducing the full quantitative model, I first describe the key mechanism. To gain intuition for how variations in growth together with labor market frictions generate the correlation between mean income growth and income growth skewness, consider the following simple model:

The economy grows at a steady rate g. At time t, an individual can either be employed or unemployed. An unemployed individual receives unemployment benefits $(1+g)^t\bar{b}$. A worker's wage stays constant until he receives a new wage offer or he becomes unemployed.

- With probability λ_u, an unemployed individual receives a wage offer. The wage offer is $(1+g)^t\bar{x}$, where $\bar{x}$ is a constant.
- With probability λ_e, an employed individual receives a wage offer, which is also $(1+g)^t\bar{x}$.
- With probability δ, the employed individual becomes unemployed.

Figure 4.1: *Intuition for why the job-ladder model generates a positive correlation between mean log income growth and log income growth skewness. In a high-growth environment, the relative wage of an individual moves much quicker toward the benefit level, which makes the positive wage-offer shocks larger and the negative unemployment shocks smaller than in a low-growth regime.*



The reasoning that follows is illustrated in Figure 4.1. Consider an individual that receives a wage-offer shock k periods after he last received a wage-offer shock. The wage-offer shock changes his wage from $(1+g)^{t-k}\bar{x}$ to $(1+g)^t\bar{x}$. That is, his log wage increases by $k\log(1+g)$.

Now, consider an individual who receives a job-separation shock k periods after he last received a wage-offer shock. The job-separation shock changes his income from $(1+g)^{t-k}\bar{x}$ to $(1+g)^t\bar{b}$. That is, his log income falls by $k\log(1+g) + \left(\log\bar{b} - \log\bar{u}\right)$.

If we compare the individual's income changes in a high-growth regime with the income changes in a low-growth regime, we see that the positive shocks (the log wage increase of $k\log(1+g)$) are higher in a high-growth regime while the absolute value of the negative shocks (the log income fall by $\left(\log\bar{x} - \log\bar{b}\right) - k\log(1+g)$) is lower in the high-growth regime.

This reasoning holds true for all k, as long as $\log\bar{x} - \log\bar{b} > k\log(1+g)$, and the result is that the skewness of the log income shock distribution will be higher (more positive) in the high growth environment than in the low growth environment. Crucial for this argument is that the wage of ongoing matches does not grow at the same rate as productivity growth.

4.2 Generalizing the Simple Example

The mechanism of the simple example is a particular instance of a more general reduced-form class of income processes which can be described as follows:

- Let an individual's log income at time t be denoted by y_t.

- With probability p, a new log income is drawn, $y_{t+1} = Y + gt$ with $Y \sim F$.
- With probability $1-p$, the $t+1$ income remains the unchanged, $y_{t+1} = y_t$.
- The econometrician observes the income with some i.i.d. noise, $\tilde{y}_t = y_t + \epsilon_t$.

In this framework, to a first order, the standard deviation does not move with the mean growth rate while the skewness does. This conclusion holds true both under comparative statics, varying the steady-state growth rate g, and in a time-series sense, studying a one-time shock to the growth rate g. The slope of the relationship between the mean and the skewness is the same both in the comparative statics and in response to a shock. The results are summarized in the following proposition, the proofs are delegated to Appendix B.

Proposition 4.1. *In the above framework,*

- *The comparative statics varying the steady-state growth rate g give the following relationship between the mean, standard deviation and skewness of the income growth distribution:*
  - *The mean and skewness of the income growth distribution satisfy the relationship*

$$\mathrm{Skew}_{\Delta y} = \mathrm{Skew}_{\Delta y}|_{g=0} + \frac{3(1-p)}{\sqrt{2}} \frac{\sigma_Y^2}{\left(\sigma_\epsilon^2 + p\sigma_Y^2\right)^{3/2}} \mathrm{Mean}_{\Delta y}$$

    to a first order in g.
  - *There is no first-order effect on the standard deviation of the income growth distribution.*
- *The immediate response of the mean, standard deviation and skewness of the income growth distribution to a shock to the growth rate g give the following relationship:*
  - *The mean and skewness of the income growth distribution satisfy the relationship*

$$\mathrm{Skew}_{\Delta y} = \mathrm{Skew}_{\Delta y}|_{g=0} + \frac{3(1-p)}{\sqrt{2}} \frac{\sigma_Y^2}{\left(\sigma_\epsilon^2 + p\sigma_Y^2\right)^{3/2}} \mathrm{Mean}_{\Delta y}$$

    to a first order in g.
  - *There is no first-order effect on the standard deviation of the income growth distribution.*

Proof. See Appendix B. □

This reduced-form class of income processes generates the salient facts from Section 3. Both in the time series and in the cross section, mean income growth and income growth skewness are closely linked, while the income growth dispersion is not. The class also satisfies that the slope of the mean-skewness relationship is identical in the time dimension and in the cross section, which is consistent with Figure 1.1.

In the reduced-form framework above, there are no choices and the comparative statics become particularly simple. In general, strategic choices and equilibrium effects can potentially mitigate the basic mechanism. Nonetheless, the qualitative feature that growth generates more positive skewness is something that we argue is likely to hold in settings with choices and equilibrium effects. The basic mechanism is consistent with many different micro settings.

One class of settings has rigid wages and productivity growth. Monopolistic unions adjusting the wages subject to a Calvo friction (as is common in the New-Keynesian literature, following Erceg et al. (2000)) and Nash bargaining with staggered wages subject to a Calvo friction (Gertler and Trigari, 2009) do generate cyclical skewness, but also cyclical dispersion in the basic set up. The reason why income dispersion is also cyclical is that income growth dispersion is second order in these frameworks, there are no micro sources of wage dispersion except the Calvo friction and aggregate shocks. In the context of our proposition, this is akin to setting $\sigma_Y = \sigma_\epsilon = 0$. To meaningfully study the cyclical fluctuation of dispersion of income growth, there needs to be nontrivial dispersion of income growth, for example by introducing firm heterogeneity or both negative and positive shocks.

Another type of settings generating the mean-skewness relationship has match-specific productivities and wages that respond one-to-one with match-specific productivity, together with the assumption that productivity growth shocks only affect future potential matches, not current ongoing matches. Wages can be flexible, as in Hagedorn and Manovskii (2013), but need not be. In the quantitative model framework below, wages are given by piece-rate contracts, which means that wages respond one-to-one with productivity despite not being flexible (the framework also includes sequential-auctioning wage setting, so wages are history dependent).

Next, I introduces the quantitative model of the labor market to investigate to what extent the qualitative insights from the reduced-form framework can quantitatively account for the cyclical and cross-sectional variations in income growth skewness.

4.3 Model

Model Framework The model framework is based on Bagger et al. (2014), who estimate a quantitative job-ladder model using Danish matched employer-employee data. In Bagger et al. (2014), workers and firms bargain over the piece rate and not the wage. In order to have wages not move one-to-one with productivity, I make the assumption that productivity growth is not reflected in the productivity of ongoing matches but only in the productivity of potential future matches. The model innovation is to introduce growth in the offer-productivity distribution.

I describe the model framework in a setting with deterministic growth. Thereafter, we do comparative statics exercises which correspond to the cross-sectional empirical result. Finally, I study the effects of stochastic shocks which corresponds to the time-series empirical result.

Time is discrete, there is a unit continuum of workers and there is a unit continuum of employment opportunities with different productivities. Wages are defined as piece-rate contracts, the worker is paid an endogenously determined share of the match output. Wage setting follows Postel-Vinay and Robin (2002) and Cahuc et al. (2006): If the worker receives an outside offer, the current and prospective future employer engage in Betrand competition for the worker's services.

Production Log output for a firm-worker match is

$$y_{it} = p_{it} + h_{it}, \qquad h_{it} = \epsilon_{it} + \alpha_i + g(e_{it})$$

where p_{it} is the match-specific productivity, ϵ_{it} is a temporary shock, modeled as an $AR(1)$, α_i is worker-specific productivity and e_{it} is the cumulative labor-market experience of the worker. $g(e_{it})$ captures the human capital of the worker as a function of experience.

Preferences The worker has log preferences and consumes his wage or unemployment benefit, with discount rate ρ.

Timing In the beginning of the period, ϵ_{it} is revealed. The worker has the option to voluntarily become unemployed. If the worker is employed, but the value of being employed is lower than the value of being unemployed, the employer increases the worker's wage so that the worker is indifferent between working and not working, and the worker remains employed. If the wage required to keep the worker employed is so high that the employer would make a loss, the worker becomes unemployed. Employed workers' experiences are updated from $e_{i,t-1}$ to $e_{it} = e_{i,t-1} + 1$.

Thereafter, production occurs. An employed worker is paid w_{it}. The process by which w_{it} is determined will be described below.

At the end of the period, with probability μ the worker leaves the sample. With probability δ the match is dissolved, the worker finds a new match immediately with probability κ, else he becomes unemployed. With probability λ_1 the worker receives an outside wage offer with match productivity p' drawn from the distribution F_t. With probability λ_0, an unemployed worker receives a wage offer, also drawing match productivity p' from the distribution F_t.

Wages Wages are defined as piece-rate contracts. If log production is y_{it}, then the worker receives log wage $w_{it} = r_{it} + y_{it}$ where $R_{it} = e^{r_{it}} \leqslant 1$ is the endogenously determined piece rate. The value of a match with productivity p and piece rate r at time t for a worker with human capital h_t is denoted $V_t(r, p, h_t)$.

If an employed worker receives an outside offer, the incumbent employer and the outside employer bargain over the worker's services. The firm that values the worker the most, the firm with the highest productivity, wins the bargaining and hires (or retains) the worker.

If the outside employer has higher productivity $p' > p_{it}$, then the outside employer wins the bargaining by offering a piece rate r' satisfying

$$E_t V_{t+1}(r', p', h_{t+1}) = E_t \{V_{t+1}(0, p_{it}, h_{t+1}) + \beta [V_{t+1}(0, p', h_{t+1}) - V_{t+1}(0, p_{it}, h_{t+1})]\}.$$

The employer and the worker share the total surplus $E_t V_{t+1}(0, p', h_{t+1}) - E_t V_{t+1}(0, p_{it}, h_{t+1})$ with the worker receiving the share β of the log surplus.[6]

If the current employer has higher productivity, $p_{it} \geqslant p'$, then the current employer increases the piece rate by

$$E_t V_{t+1}(r', p_{it}, h_{t+1}) = E_t \{V_{t+1}(0, p', h_{t+1}) + \beta [V_{t+1}(0, p_{it}, h_{t+1}) - V_{t+1}(0, p', h_{t+1})]\}$$

unless p' is so low that the above would result in a lowering of the piece rate.

The value of being unemployed $V_{0,t}$ is equal to employment in the least productive firm. If the worker is unemployed and receives an offer, then the worker is employed with piece rate given by

$$E_t V_{t+1}(r', p', h_{t+1}) = E_t \{V_{0,t}(h_t) + \beta [V_{t+1}(0, p', h_{t+1}) - V_{0,t}(h_t)]\}$$

The Offer-Productivity Distribution The offer-productivity distribution F_t grows at a rate g, generating exponential growth,

$$F_t(p) = F(p - gt)$$

where F is the time-invariant detrended offer-productivity distribution. F has support $[\tilde{p}_{\min}, \tilde{p}_{\max}]$. Therefore, F_t has support $[\tilde{p}_{\min} + gt, \tilde{p}_{\max} + gt]$.

[6] β is strictly speaking not a Nash bargaining weight since the workers have logarithmic utility and the firms should be risk neutral. Cahuc et al. (2006) work out a modified Rubinstein bargaining protocol that rationalizes β as the bargaining weight.

Value Functions The value of an employed worker is given by

$$
\begin{aligned}
V_t(r,p,h_t) = \max \Bigg\{ & V_t(0, \tilde{p}_{\min} + gt, h_t), \\
& r + p + \frac{\delta(1-\kappa)}{1+\rho} V_{0,t}(h_t) \\
& + \frac{\delta\kappa}{1+\rho} \int_{\tilde{p}_{\min}+gt}^{\tilde{p}_{\max}+gt} E_t \left[(1-\beta)V_{0,t}(h_t) + \beta V_{t+1}(0,x,h_{t+1})\right] dF_t(x) \\
& + \frac{\lambda_1}{1+\rho} \int_{p}^{\tilde{p}_{\max}+gt} E_t \left[(1-\beta)V_{t+1}(0,p,h_{t+1}) + \beta V_{t+1}(0,x,h_{t+1})\right] dF_t(x) \\
& + \frac{\lambda_1}{1+\rho} \int_{q_t(r,p,h_t)}^{p} E_t \left[(1-\beta)V_{t+1}(0,x,h_{t+1}) + \beta V_{t+1}(0,p,h_{t+1})\right] dF_t(x) \\
& + \frac{1}{1+\rho} \left[1 - \mu - \delta - \lambda_1 \bar{F}_t(q_t(r,p,h_t))\right] E_t V_{t+1}(r,p,h_{t+1}) \Bigg\}
\end{aligned}
$$

where $q_t(r,p,h_t)$ is implicitly defined as the threshold for which the offer is so low that it does not bid up the worker's piece rate. Since the worker always has the option to be unemployed, with value $V_t(0, \tilde{p}_{\min} + gt, h_t)$, the recursive formulation contains a max operator.

Solving the Model It is possible to reformulate the problem in terms of detrended variables. Introducing a non-zero growth rate in the offer distribution, we lose analytical tractability but not computational tractability. The solution method uses that we can solve easily for $\partial V/\partial p(0,p)$ by first integrating by parts. When $g = 0$ we can solve directly for $\partial V/\partial p(0,p)$, when $g \neq 0$ we instead prove that $\partial V/\partial p(0,p)$ is the unique fixed point of a contraction mapping, and compute it by repeatedly applying the contraction mapping.

The details of the solution are described in Appendix C.

Parameter Values I take parameter estimates from Bagger et al. (2014), who estimate a model of this form (with $g = 0$). They estimate the model using indirect inference. The model moments they match to data are:

1. The survivor functions for unemployment-to-employment transitions, employment-to-unemployment transitions and job-to-job transitions.

2. The tenure and experience profiles from a Mincer regression of log wage on a cubic in tenure, a cubic in experience as well as worker and firm fixed effects. They also target the distribution of firm and worker fixed effects, as well as the autocovariance structure of the error term.

3. The within-job wage growth profile from a within-job regression of log wage growth on a cubic in tenure. They also target the standard deviation, skewness and kurtosis of the residuals, as well as the autocovariance of the residuals.

They also target some aggregate statistics, as well as firm-level value added. They stratify their sample by years of education into three groups. We use their parameter values for the middle sample with 12-14 years of schooling.

The monthly discount rate ρ is set to 0.005 and the monthly attrition rate to 0.0018. The offer-productivity distribution follows a Weibull distribution, $F(p) = 1 - \exp(-[\nu_1(p - \nu_0)]^{\nu_2})$ with $\nu_0 = p_{\min} = 4.92$, $\nu_1 = 9.00$, $\nu_2 = 0.70$. The transitory productivity shock ϵ_{it} follows an $AR(1)$ with monthly autoregressivity $\eta = 0.70$ and shock standard deviation $\sigma_u = 0.10$. The workers' bargaining power is $\beta = 0.30$. The experience accumulation function is a cubic in experience, $g(e) = 0.0136e - 0.00039e^2 + 0.0000017e^3$. The monthly job-separation rate δ is 0.0072.[7]

Bagger et al. (2014) estimate heterogenous job-finding rates. In the estimation, they link the job-finding rate heterogeneity to person-specific productivity α. The estimated job-finding rate from unemployment is

$$\lambda_0(\alpha) = \frac{\theta_0(\alpha)}{1 + \theta_0(\alpha)} \qquad\qquad \theta_0(\alpha) = \exp\left[-0.94 + 24.47\alpha\right]$$

and the estimated offer-arrival rate from employment is

$$\lambda_1(\alpha) = \frac{\theta_1(\alpha)}{1 + \theta_1(\alpha)} \qquad\qquad \theta_1(\alpha) = \exp\left[-2.62 + 2.36\alpha\right].$$

The estimated job-finding probability conditional on a job-separation shock is

$$\kappa(\alpha) = \frac{\theta_2(\alpha)}{1 + \theta_2(\alpha)} \qquad\qquad \theta_2(\alpha) = \exp\left[0.41 + 6.25\alpha\right].$$

For the median worker ($\alpha = 0$), the monthly job-finding rate from unemployment is 0.28, the monthly offer-arrival rate from employment is 0.07, and the job-finding probability conditional on a job-separation shock is 0.60.

The variation in worker productivity α is normally distributed with $\sigma_\alpha = 0.09$. In the computations, we discretize the distribution of α with Gauss-Hermite quadrature using 7 gridpoints.

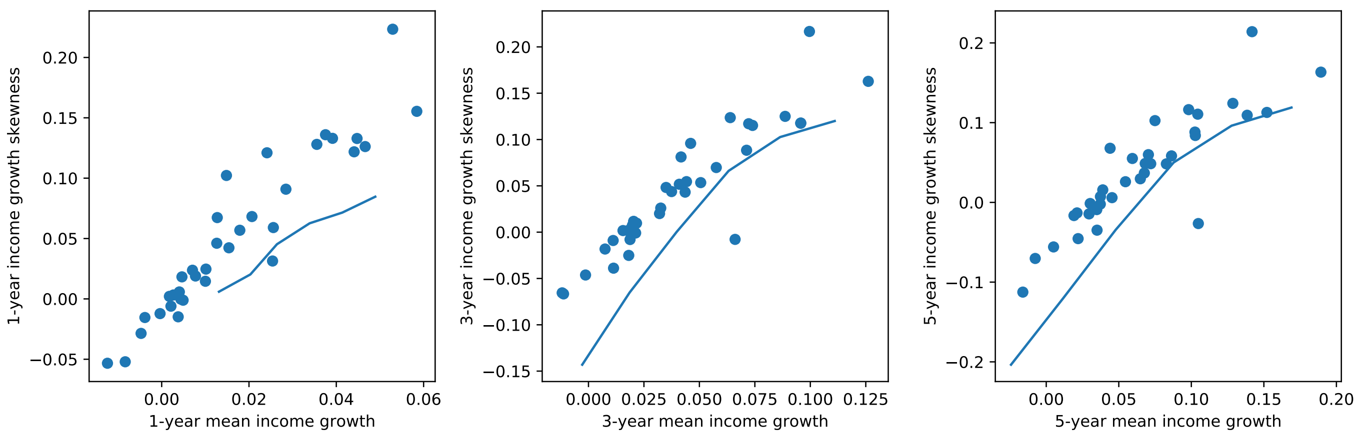
4.4 Comparative Statics – Varying the Growth Rate

Now we turn to the first model experiment. We vary the growth rate g from -0.03 to 0.02. For each g, we simulate a population of 10,000 individuals for thirty years, corresponding to a life cycle of work[8] and compute the mean income growth as well as the income growth skewness. Annual income is computed by aggregating the monthly incomes, with the individuals earning zero labor income while unemployed. When

[7] Throughout, we refer to the probabilities of job separation, offer arrival and job finding as rates, although strictly speaking rates refer to continuous-time arrival rates.

[8] There is a tension between our empirical work, in which we studied workers aged 25-59, and the estimation of Bagger et al. (2014) who followed workers for a life cycle of 30 years.

Figure 4.2: *The cross-sectional relationship between mean income growth and income growth Kelley's skewness. The dots show the relationship of the mean income growth and income growth Kelley's skewness for 35 of the 37 educational categories. The curves depict the comparative statics for mean income growth and income growth Kelley's skewness of varying* g *from* -0.03 *to* 0.02.



computing the model income growth moments, observations with zero annual income in either t or $t+k$ are removed, as in our empirical analysis. Individuals who exit the sample (receive a μ shock) are also removed from the sample.

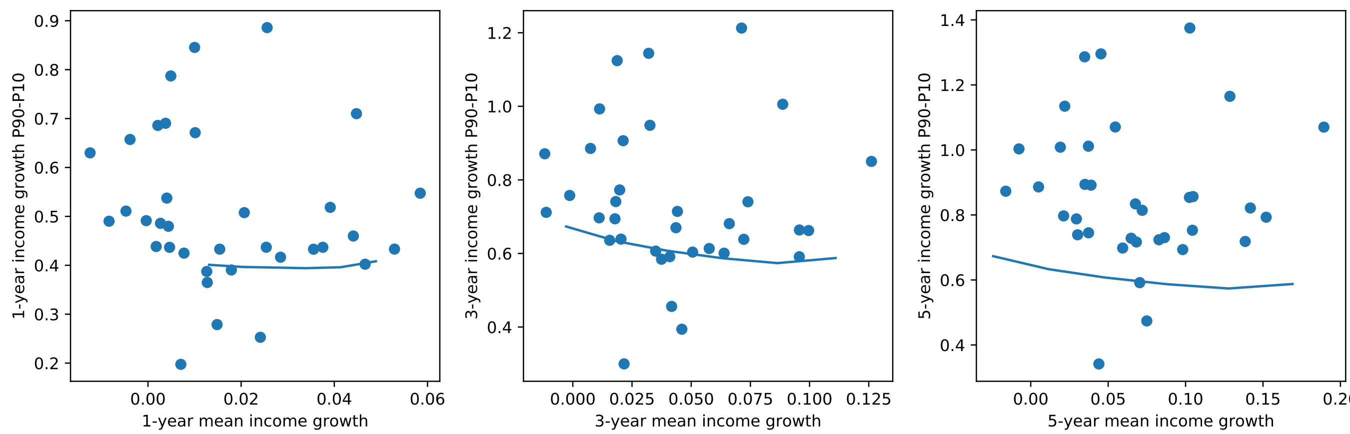
In Figure 4.2, we plot the comparative statics for the mean income growth rate and the income growth Kelley's skewness at the one-year, three-year, and five-year horizons together with the cross-sectional distribution across educational categories. In the figure, we removed two outliers from the data points. The model misses somewhat at the one-year horizon, but provides a surprisingly good fit at both the three-year and five-year horizon. The slope of the mean-skewness relationship of the comparative statics is close to identical to the cross-sectional slope, and the level only undershoots slightly.

The model also succesfully captures the absence of link between the mean income growth and P90-P10 dispersion of income growth. In Figure 4.3, we display the relationship between the mean income growth and income growth P90-P10 dispersion at the different horizons. We note that the model captures the level of dispersion, although the dispersion at the five-year horizon is somewhat at the lower end of the spectrum.

In contrast, varying the job-separation rate δ, the job-finding rate λ_0 and the on-the-job offer arrival rate λ_1 fail to capture the cross-sectional relationship. In Figure 4.4, we show the comparative statics of varying δ from 0.003 to 0.027 (roughly half and four times the estimated $\delta = 0.0072$). Even quadrupling the job-separation rate is not enough to generate sufficient variation in the mean income growth. Furthermore, for high job-separation rates, skewness is increasing in the job-separation rate.

In Figure 4.5, we vary the constant term in the expression for the job-finding rate such that the median

Figure 4.3: The cross-sectional relationship between mean income growth and income growth P90-P10 dispersion. The dots are 35 of the 37 educational categories. The curves depict the comparative statics of varying g *from* -0.03 *to* 0.02.



job-finding rate ranges from a low 0.05 to a high 0.88. The variation in the job-finding rate can generate a very high positive skewness, but does not generate much growth together with the skewness. For low values of the job-finding rate, skewness does not go below 0.

In Figure 4.6, we vary the constant term in the expression for the on-the-job offer arrival rate such that the median offer-arrival rate ranges from a low 0.03 to a high 0.12. The variation in the on-the-job offer arrival rate generates a negative relationship between the mean income growth and the income growth skewness at the three-year and five-year horizons.

I therefore conclude from the comparative statics exercise that variations in growth rates can account for the cross-sectional relationship between mean income growth and income growth skewness while the standard labor market parameters cannot.

4.5 Time-Series Relationship – Stochastic Growth Rate

Next, we study the time-series relationship between the mean income growth and the skewness by computing impulse responses for the mean income growth and income growth skewness. To ease the computational cost, we restrict the heterogeneity (all households have $\alpha = 0$) and turn off human-capital accumulation ($h_{it} = 0$). It is unlikely that heterogeneity or human-capital accumulation play a quantitative role in the relative responsiveness of mean income growth and income growth skewness to shocks.

We consider a shock to the growth rate of the offer distribution and a shock to the job-separation rate, both with autoregressivity of 0.93 at the monthly frequency. The growth shock's magnitude is 0.02 in annualized growth. The job-separation rate shock is a lowering of the job-separation rate by 0.03/12, a

Figure 4.4: *The cross-sectional relationship between mean income growth and income growth Kelley's skewness. The dots are 35 of the 37 educational categories. The curves depict the comparative statics of varying the job-separation rate* δ *from* 0.003 *to* 0.027.

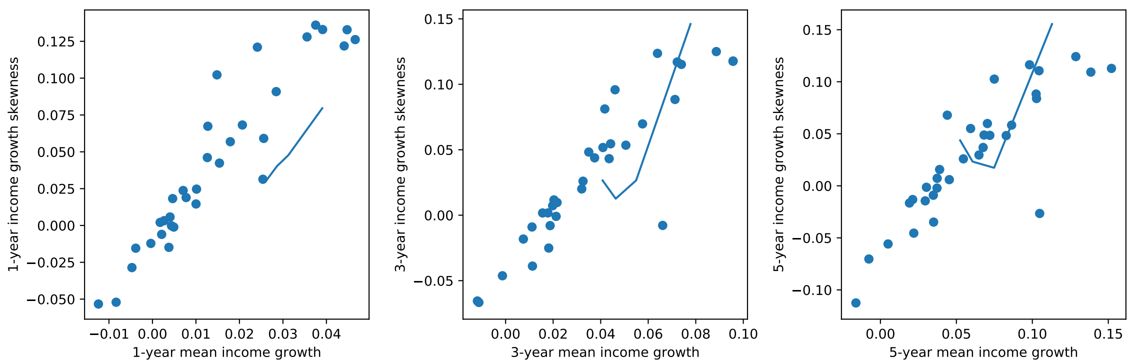


Figure 4.5: *The cross-sectional relationship between mean income growth and income growth Kelley's skewness. The dots are 35 of the 37 educational categories. The curves depict the comparative statics of varying the job-finding rate* λ_0 *such that the median job-finding rate ranges from* 0.05 *to* 0.88.

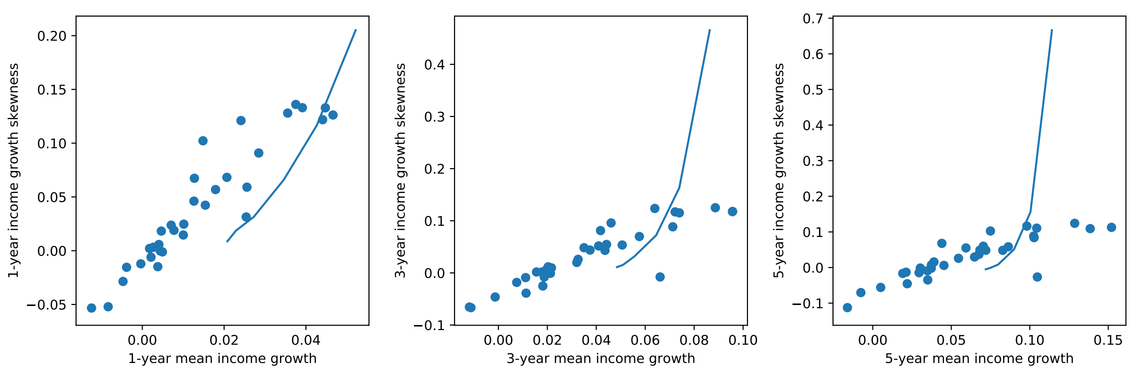


Figure 4.6: *The cross-sectional relationship between mean income growth and income growth Kelley's skewness. The dots are 35 of the 37 educational categories. The curves depict the comparative statics of varying the on-the-job offer arrival rate* λ_1 *such that the median on-the-job offer arrival rate ranges from* 0.03 *to* 0.12.

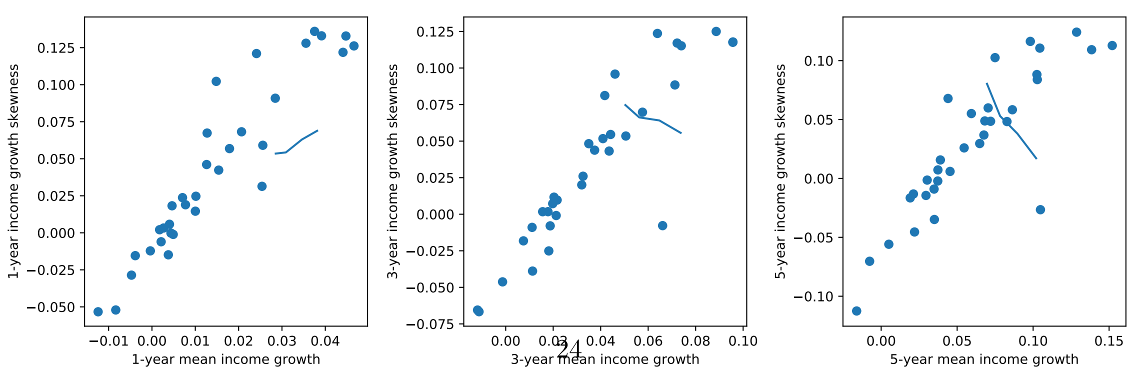
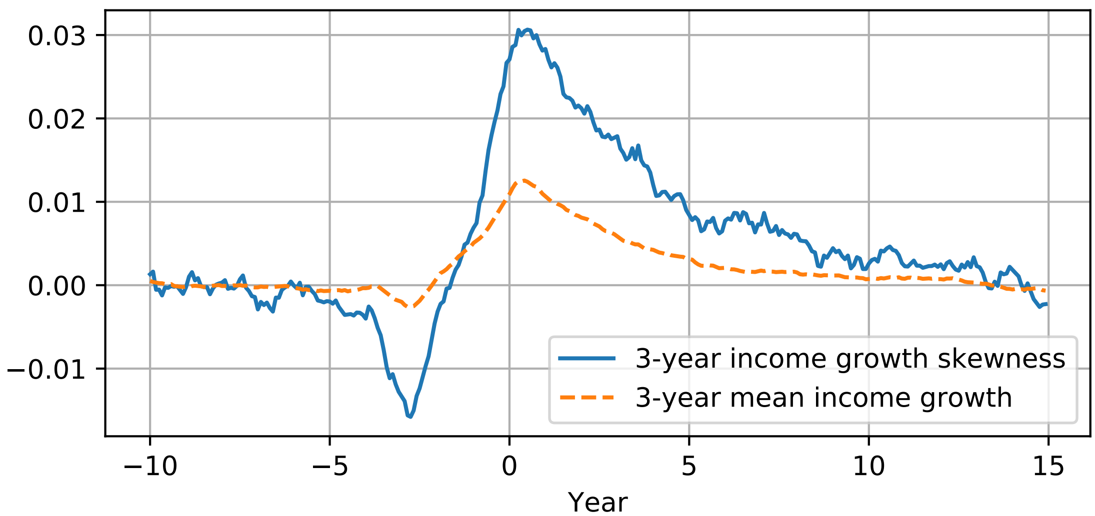


Figure 4.7: *Impulse responses for three-year mean income growth and three-year income growth Kelley's skewness. The shock to the offer-distribution growth occurs at time* 0*.*



lowering of the job-separation rate by 35 percent. The magnitudes of the shocks are chosen to generate responses of roughly similar magnitudes.

In Figure 4.7, we plot the impulse responses of three-year income growth skewness and three-year mean income growth to a growth shock, based on simulating 1,000,000 individuals. Three years before the shock, the three-year moments begin to move. Initially, there is a fall in income. This is because on impact, some households choose to leave their current job to search for a new and better job full time. After the initial impact, both the mean income growth and the skewness increase, with skewness at peak reaching roughly 0.03 while the growth peaks at 0.01.

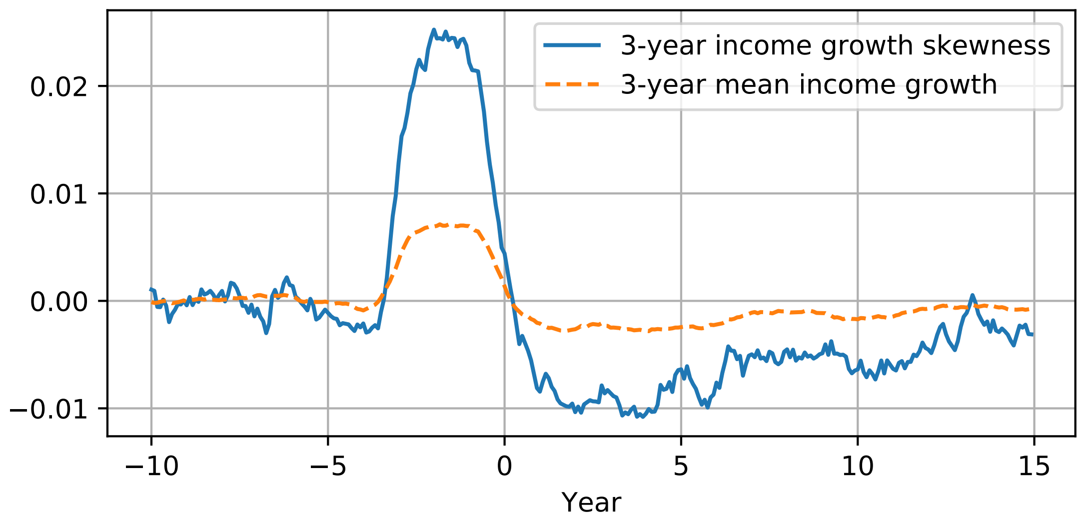
In Figure 4.8, we plot the impulse responses of three-year income growth skewness and three-year mean income growth to a job-separation rate shock, based on simulating 1,000,000 individuals. The fall in the job-separation rate shock generates an increase in the skewness at peak of approximately 0.025 while the mean income growth increases by 0.75.

4.5.1 Inferring Moments from the Impulse-Response Functions

As pointed out by Boppart et al. (2017), impulse-response functions are sufficient statistics for a range of moments. In particular, from the impulse-response functions we can infer the variance and correlation of the variables.

The linear impulse-response function for a variable X is a mapping $(\epsilon, k) \mapsto \mathrm{irf}_{x,k}\epsilon$, linear in ϵ, that

Figure 4.8: *Impulse responses for three-year mean income growth and three-year income growth Kelley's skewness. The shock to the job-separation rate occurs at time* 0.



maps a shock ϵ to the response of variable X at time k after the shock (or before, if $k < 0$). If the only shock in the system is ϵ, the value of X_t can be written in terms of previous shocks on the form

$$X_t = \sum_k \mathrm{irf}_{x,k} \epsilon_{t-k}.$$

The variance of X is given by $\sigma_\epsilon^2 \sum_k \mathrm{irf}_{x,k}^2$. Therefore, the relative volatility of the income growth skewness, in relation to the volatility of mean income growth is given by

$$\frac{\sigma(\mathrm{skew})}{\sigma(\mathrm{mean})} = \sqrt{\frac{\sum_k \mathrm{irf}_{\mathrm{skew},k}^2}{\sum_k \mathrm{irf}_{\mathrm{mean},k}^2}}$$

where $\mathrm{irf}_{\mathrm{skew}}$ and $\mathrm{irf}_{\mathrm{mean}}$ are the impulse responses of the skewness and mean respectively.

Analogously, the correlation of the skewness and mean is given by

$$\rho_{\mathrm{mean},\mathrm{skew}} = \frac{\sum_k \mathrm{irf}_{\mathrm{mean},k} \mathrm{irf}_{\mathrm{skew},k}}{\sqrt{\sum_k \mathrm{irf}_{\mathrm{mean},k}^2 \sum_k \mathrm{irf}_{\mathrm{skew},k}^2}}.$$

The underlying assumption for the existence of an impulse-response function is that the model is locally linear. However, there is a discontinuity with respect to the sign of the shock to the growth rate in our model framework. Initially, in response to a positive shock, some workers voluntarily become unemployed. This

Table 4.1: *Moments in an environment with only growth shocks. The correlation between the mean income growth and the income growth skewness at the one-year, three-year, and five-year horizons, as well as the relative volatility of the skewness compared with the mean income growth. The data columns are the averages of the moments for the 37 different educational categories.*

	Corr.	Corr (data)	Std(skew)/std(mean)	Std(skew)/std(mean) (data)
1-year income growth	0.90	0.87	2.63	4.14
3-year income growth	0.97	0.88	2.69	2.78
5-year income growth	0.95	0.88	1.79	2.12

Table 4.2: *Moments in an environment with only job-separation shocks. The correlation between the mean income growth and the income growth skewness at the one-year, three-year, and five-year horizons, as well as the relative volatility of the skewness compared with the mean income growth. The data columns are the averages of the moments for the 37 different educational categories.*

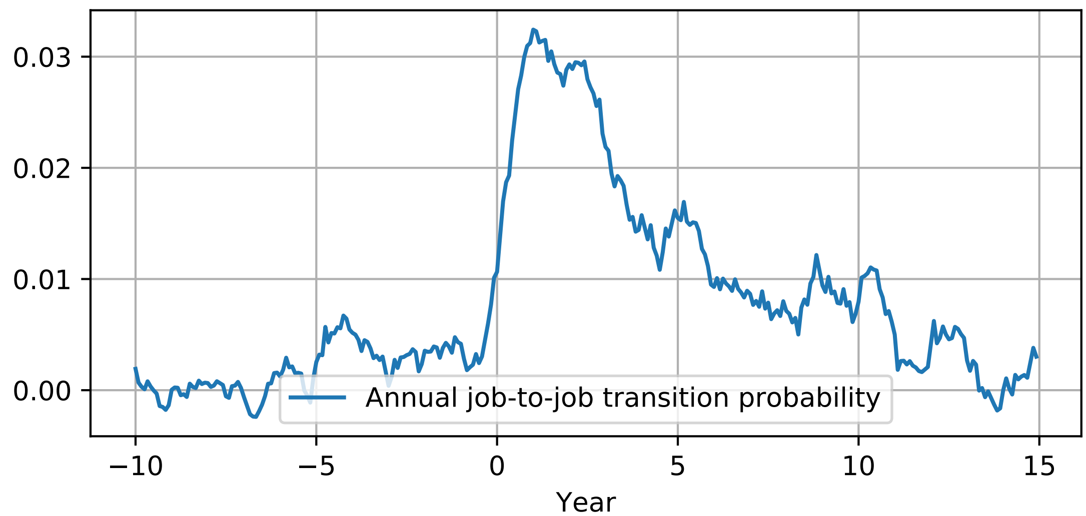
	Corr.	Corr (data)	Std(skew)/std(mean)	Std(skew)/std(mean) (data)
1-year income growth	0.92	0.87	2.95	4.14
3-year income growth	0.99	0.88	3.46	2.78
5-year income growth	0.93	0.88	2.15	2.12

response has no counterpart for negative shocks, workers cannot voluntarily become employed. Nonetheless, we take the computed impulse responses as impulse-response functions and compute the moments. As robustness, we also study the impulse response to a negative growth shock.

In Table 4.1, the correlation between the model skewness and model mean are shown, together with a measure of the relative volatility of the skewness compared to the mean. The model correlations for the three-year and five-year horizons are higher than the average correlation in the data. The relative magnitude of the volatility of the skewness compared to the mean is very close to the empirical counterpart, at both the three-year and five-year horizons. As in the comparative-statics exercise, a growth rate shock does not capture quantitatively the movements at the one-year horizon. In Appendix A, Figure A.3, we show the moments computed using a negative impulse response. The results are virtually the same. At no point was the model calibrated to match any moments directly related to the cyclicality of the skewness, so we regard this as a success of the model.

Shocks to the job-separation rate generate broadly similar correlations and relative volatility of the skewness, as shown in Table 4.2. The model driven by job-separation rate shocks overstates the relative volatility of three-year income growth by 24 percent, but matches the five-yer relative volatility well. Just as for the growth shock, it misses the one-year relative volatility.

Figure 4.9: *Impulse-response function for job-to-job transition rate, normalized (so peak at three percent). The shock to the growth rate occurs at time* 0*.*

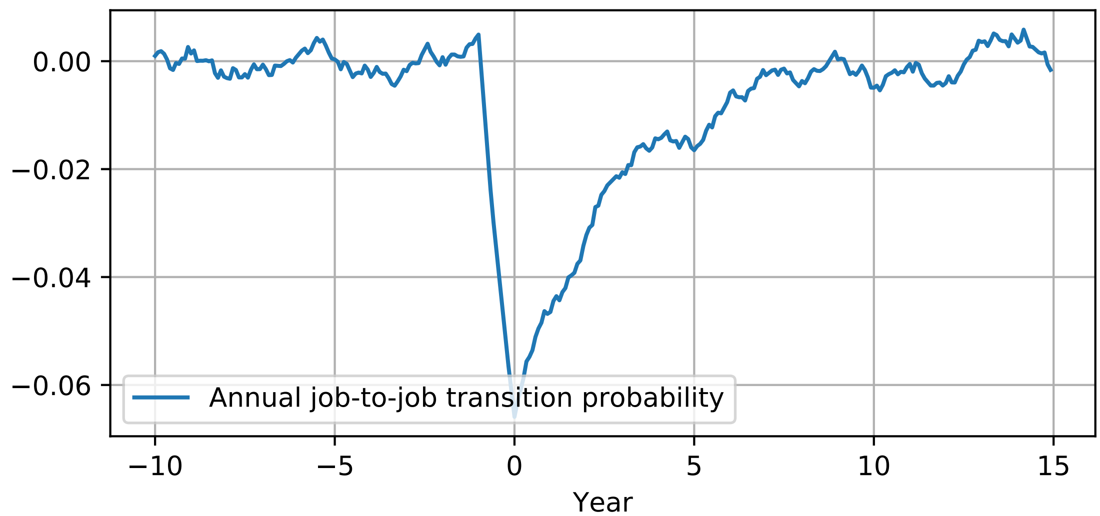


4.5.2 The Implications for the Job-to-Job Transition Rate

The growth shock and the job-separation rate shock have differential implications for the job-to-job transition rate. A growth shock increases the share of job offers that lead to a job change and therefore increases the job-to-job transition rate. In contrast, a negative job-separation rate shock lowers the share of workers who were recently unemployed and are climbing up the ladder. It therefore decreases the job-to-job transition rate. In Figure 4.9 and Figure 4.10, we show the impulse response functions for the growth and job-separation rate shocks respectively.

A recent literature has documented that the job-to-job transition rate moves with wage growth. Faberman and Justiniano (2015) document using US data that the aggregate job-to-job transition rate is highly correlated with different measures of wage growth. Moscarini and Postel-Vinay (2016) run a regression of both the unemployment-to-employment transition rate and the employment-to-employment transition rate on a wage growth measure and find that the regression model only puts weight on the employment-to-employment transition rate for predicting wage growth. Moscarini and Postel-Vinay (2017) do a two-stage exercise. First, they run a regression with a total of 17,600 age-education-race-gender-month dummies on labor market transitions and earnings/wage growth (with some additional controls). This gives age-education-race-gender-month specific employment-to-employment labor market transition rates as well as an age-education-race-gender-month specific earnings/wage growth. In the second step, they run a regression for the 17,600 categories of the estimated labor market transitions (between the three states employment,

Figure 4.10: *Impulse-response function for job-to-job transition rate, normalized (so trough at negative six percent). The shock to the job-separation rate occurs at time* 0*.*



non-employment and unemployment, as well as employment-to-employment) and find that there is a positive relationship between the employment-to-employment transition rate and the nominal wage growth, controlling for the other labor market transition rates and the unemployment rate. Karahan et al. (2017) run a regression of the unemployment-to-employment job-finding rate and the employment-to-employment job-finding rate on labor income, using US state-level data at the quarterly frequency and find that higher employment-to-employment job-finding rates are associated with higher labor income.

This evidence leads us to argue that a good theory of the comovement of mean income growth and income growth skewness should generate positive comovement of mean income growth and job-to-job transitions. Job-separation rate shocks generate a negative correlation of mean income growth and job-to-job transitions while productivity growth shocks generate positive comovement of mean income growth, income growth skewness, and job-to-job transitions simultaneously.

5 Concluding Remarks

The connection between mean income growth and the skewness of the income growth distribution is surprisingly tight both in the time-series dimension, in the cross section between educational categories, and in the time-series dimension for the different educational categories.

The high correlation both in the time dimension and the cross section suggests an underlying deeper mechanism and I provide one potential such mechanism: Labor-market frictions together with variations in productivity growth generate the mean-skewness relationship. Using a reduced-form framework I show that productivity growth variations generate first-order variations in skewness but only second-order variations in dispersion.

In a sequential-auctioning job-ladder model, this mechanism quantitatively accounts for the mean-skewness relationship both in the cross section and in the time dimension. In contrast, variations in job-finding rates, job-separation rates and on-the-job offer arrival rates do not capture the mean-skewness relationship. Furthermore, job-separation rate shocks generate a counterfactual negative correlation of the job-to-job transition rate and mean income growth.

In conclusion, I provide an intuitive mechanism for variations in income growth skewness, and show with an externally estimated model that the mechanism quantitatively accounts for the variations in income growth skewness.

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A Additional Tables and Figures

Figure A.1: *Statistical skewness of the labor income growth (blue solid, left) comoves with the mean of labor income growth (red dashed). The standard deviation (blue solid, right) does not.*

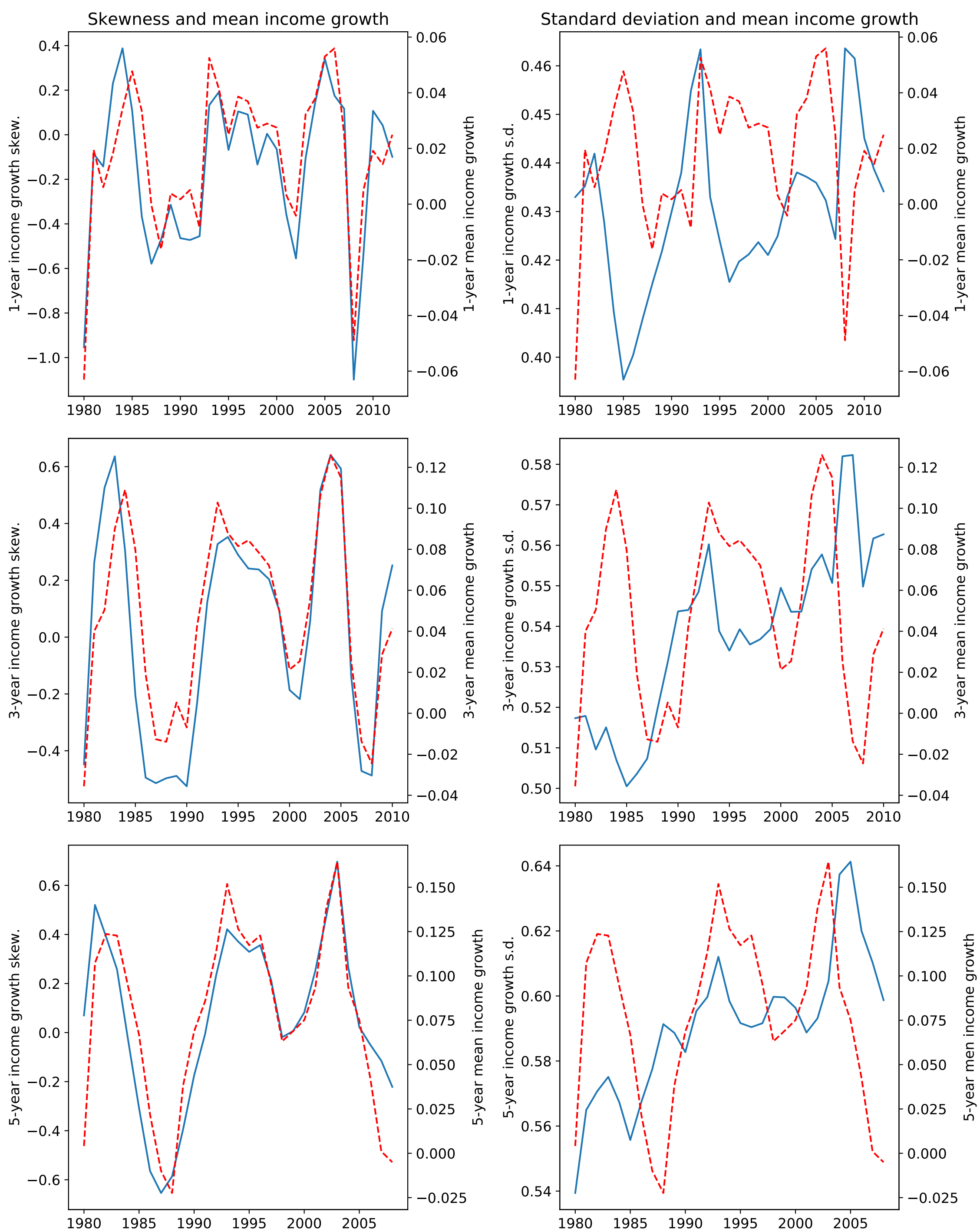

Table A.1: *Full list of the 37 educational categories*

Educational field	Educational level
Generic programmes and qualification, Basic programmes and qualifications	General lower secondary education (insufficient for partial level completion)
Generic programmes and qualification, Basic programmes and qualifications	General lower secondary education (completed, with access to upper secondary education)
Generic programmes and qualification, Basic programmes and qualifications	General upper secondary education (completed, with access to tertiary education)
Education, Teacher training without subject specialization	Professional Bachelor's, First degree (3-4 years)
Arts, Audio-visual techniques and media production	Vocational upper secondary education (completed, without direct access to tertiary education)
Business and administration, General	Academic Bachelor's, First degree (3-4 years)
Business and administration, Accounting and taxation	Professional Bachelor's First degree (3-4 years)
Business and administration, Finance, banking and insurance	Vocational upper secondary education (completed, without direct access to tertiary education)
Business and administration, Secretarial and office work	Vocational upper secondary education (completed, with direct access to tertiary education)
Business and administration, Wholesale and retail sales	Vocational upper secondary education (completed, without direct access to tertiary education)
Business and administration, Wholesale and retail sales	Vocational upper secondary education (completed, with direct access to tertiary education)
Law, Law	Academic Master's degree following a Bachelor's degree
Engineering and engineering trades, General	Academic Master's degree following a Bachelor's degree
Engineering and engineering trades, Electricity and energy	Vocational upper secondary education (completed, with direct access to tertiary education)
Engineering and engineering trades, Electricity and energy	Vocational upper secondary education (completed, with direct access to tertiary education)
Engineering and engineering trades, Electronics and automation	Vocational upper secondary education (completed, with direct access to tertiary education)
Engineering and engineering trades, Mechanics and metal trades	Vocational upper secondary education (completed, without direct access to tertiary education)
Engineering and engineering trades, Mechanics and metal trades	Vocational upper secondary education (completed, with direct access to tertiary education)
Engineering and engineering trades, Mechanics and metal trades	Vocational upper secondary education (completed, with direct access to tertiary education)
Engineering and engineering trades, Motor vehicles, ships and aircraft	Vocational upper secondary education (completed, without direct access to tertiary education)
Manufacturing and processing, Food processing	Vocational upper secondary education (completed, with direct access to tertiary education)
Manufacturing and processing, Materials (glass, paper, plastic and wood)	Vocational upper secondary education (completed, with direct access to tertiary education)
Manufacturing and processing, Interdisciplinary	Vocational upper secondary education (completed, with direct access to tertiary education)
Architecture and construction, General	Professional Bachelor's First degree (3-4 years)
Architecture and construction, Architecture and town planning	Academic Master's degree following a Bachelor's degree
Architecture and construction, Building and civil engineering	Vocational upper secondary education (completed, without direct access to tertiary education)
Architecture and construction, Building and civil engineering	Vocational upper secondary education (completed, with direct access to tertiary education)
Architecture and construction, Building and civil engineering	Professional Bachelor's First degree (3-4 years)
Agriculture, Crop and livestock production	Vocational upper secondary education (insufficient for partial level completion)
Agriculture, Crop and livestock production	Vocational upper secondary education (completed, without direct access to tertiary education)
Agriculture, Horticulture	Vocational upper secondary education (completed, with direct access to tertiary education)
Health, Medicine	Academic Master's degree following a Bachelor's degree
Welfare, Child care and youth services	Professional Bachelor's First degree (3-4 years)
Personal services, Hotel, restaurants and catering	Vocational upper secondary education (completed, with direct access to tertiary education)
Security services, Protection of persons and property	Professional Bachelor's First degree (3-4 years)
Transport services, Transport services	Professional Bachelor's First degree (3-4 years)

Table A.2: *Regression coeffecents for regressing mean income growth on the Kelley's skewness, skewness, P90-P10 dispersion and standard deviation of income growth. Standard errors are computed using Newey-West with one lag. The coefficients for P90-P10 dispersion and standard deviation are not significant at the ten percent level. The coefficients for Kelley's skewness and skewness are significant at the* 10^{-10} *level.*

	1-year income growth	3-year income growth	5-year income growth
Kelley's skewness	4.02***	2.76***	2.09***
	(0.30)	(0.20)	(0.21)
Skewness	11.80***	7.56***	5.87***
	(0.94)	(0.55)	(0.81)
P90-P10 dispersion	-0.29	0.02	0.17
	(0.29)	(0.21)	(0.12)
Standard deviation	-0.16	0.01	0.07
	(0.13)	(0.11)	(0.08)

Table A.3: *Moments in an environment with only growth shocks, computed using the impulse response of a negative shock to the growtht rate. The correlation between the mean income growth and the income growth skewness at the one-year, three-year, and five-year horizons, as well as the relative volatility of the skewness compared with the mean income growth. The data columns are the averages of the moments for the 37 different educational categories.*

	Corr.	Corr (data)	Std(skew)/std(mean)	Std(skew)/std(mean) (data)
1-year income growth	0.94	0.87	2.84	4.14
3-year income growth	0.99	0.88	2.84	2.78
5-year income growth	0.98	0.88	1.94	2.12

B Proof for Generalizing the Simple Example

Proposition B.1. *In the above framework,*

- *The comparative statics varying the steady-state growth rate* g *give the following relationship between the mean, standard deviation and skewness of the income growth distribution:*
  - *The mean and skewness of the income growth distribution satisfy the relationship*

$$\mathsf{Skew}_{\Delta y} = \mathsf{Skew}_{\Delta y}|_{g=0} + \frac{3(1-p)}{\sqrt{2}} \frac{\sigma_Y^2}{\left(\sigma_\epsilon^2 + p\sigma_Y^2\right)^{3/2}} \mathsf{Mean}_{\Delta y}$$

    to a first order in g.
  - *There is no first-order effect on the standard deviation of the income growth distribution.*
- *The immediate response of the mean, standard deviation and skewness of the income growth distribution to a shock to the growth rate* g *give the following relationship:*
  - *The mean and skewness of the income growth distribution satisfy the relationship*

$$\mathsf{Skew}_{\Delta y} = \mathsf{Skew}_{\Delta y}|_{g=0} + \frac{3(1-p)}{\sqrt{2}} \frac{\sigma_Y^2}{\left(\sigma_\epsilon^2 + p\sigma_Y^2\right)^{3/2}} \mathsf{Mean}_{\Delta y}$$

    to a first order in g.
  - *There is no first-order effect on the standard deviation of the income growth distribution.*

Proof. We show the relationshp first for the comparative statics, then for the one-off shock.

- The steady state detrended distribution of y is $p\sum_{k\geqslant 0}(1-p)^k F(x+kg)$. The income growth variance is

$$\begin{aligned}\sigma_{\Delta\epsilon}^2 + (1-p)g^2 + p^2(\sigma_{\Delta Y}^2 + g^2) + p^2(1-p)(\sigma_{\Delta Y}^2 + (2g)^2) + \ldots =\\ (1-p)g^2 + p\sigma_{\Delta Y}^2 + p^2 g^2 \sum_{k\geqslant 0} k^2(1-p)^k =\\ = \sigma_{\Delta\epsilon}^2 + p\sigma_{\Delta Y}^2 + 2\frac{1-p}{p}g^2\end{aligned}$$

  so the standard deviation is $\sqrt{\sigma_{\Delta\epsilon}^2 + p\sigma_{\Delta Y}^2 + 2\frac{1-p}{p}g^2}$ which is second-order in g.

  Using that $E[(\Delta Y + \Delta\epsilon + kg)^3] = m_{3,\Delta Y} + m_{3,\Delta\epsilon} + 3kg(\sigma_{\Delta Y}^2 + \sigma_{\Delta\epsilon}^2) + k^3 g^3$, we compute the third moment:

$$m_3 = (1-p)\left[m_{3,\Delta\epsilon} - 3\sigma^2_{\Delta\epsilon} g - g^3\right] + p^2 \sum_{k\geqslant 0} (1-p)^k \left[m_{3,\Delta Y} + m_{3,\Delta\epsilon} + 3kg(\sigma^2_{\Delta Y} + \sigma^2_{\Delta\epsilon}) + k^3 g^3\right] =$$

$$= (1-p)\left[m_{3,\Delta\epsilon} - 3\sigma^2_{\Delta\epsilon} g - g^3\right] + p(m_{3,\Delta Y} + m_{3,\Delta\epsilon}) + 3g(\sigma^2_{\Delta Y} + \sigma^2_{\Delta\epsilon})(1-p) + p^2 \sum_{k\geqslant 0} k^3 (1-p)^k g^3.$$

To a first order in g, the skewness of the change distribution is given by

$$\frac{m_{3,\Delta\epsilon} + pm_{3,\Delta Y} + 3(1-p)\sigma^2_{\Delta Y} g}{\left(\sigma^2_{\Delta\epsilon} + p\sigma^2_{\Delta Y}\right)^{3/2}}$$

or

$$Skew_{\Delta y} = Skew_{\Delta y}|_{g=0} + \frac{3(1-p)}{\sqrt{2}} \frac{\sigma^2_Y}{\left(\sigma^2_\epsilon + p\sigma^2_Y\right)^{3/2}} Mean_{\Delta y}$$

- In response to a shock, the mean growth is pg.

  The variance is

$$(1-p)E[(\Delta\epsilon - pg)^2] + pE[(\Delta\epsilon + \Delta Y + g - pg)^2] = (1-p)(\sigma^2_\epsilon + p^2 g^2) + p(\sigma^2_\epsilon + \sigma^2_Y + (1-p)^2 g^2) =$$
$$\sigma^2_\epsilon + p\sigma^2_Y + (1-p)pg^2$$

  so the standard deviation $\sqrt{\sigma^2_\epsilon + p\sigma^2_Y + (1-p)pg^2}$ is second-order in g and the centralized third moment is

$$m_3 = E[(\Delta\tilde{y} - pg)^3] = (1-p)E\left[(\Delta\epsilon - pg)^3\right] + pE[(\Delta Y + \Delta\epsilon + (1-p)g)^3] =$$
$$= (1-p)\left[m_{3,\Delta\epsilon} - 3\sigma^2_{\Delta\epsilon} pg - (pg)^3\right] + p\left[m_{3,\Delta\epsilon} + m_{3,\Delta Y} + 3(\sigma^2_{\Delta\epsilon} + \sigma^2_{\Delta Y})(1-p)g + (1-p)^3 g^3\right] =$$
$$= m_{3,\Delta\epsilon} + pm_{3,\Delta Y} + 3(1-p)p\sigma^2_{\Delta Y} g + ((1-p)p^3 + p(1-p)^3)g$$

  Therefore, the initial response of skewness, to a first order in g, is

$$Skew_{\Delta y} = Skew_{\Delta y}|_{g=0} + \frac{3(1-p)}{\sqrt{2}} \frac{\sigma^2_Y}{\left(\sigma^2_\epsilon + p\sigma^2_Y\right)^{3/2}} Mean_{\Delta y}.$$

  Note that $Mean_{\Delta y} = pg$.

$\square$

C Model Solution

Value Functions First, we rewrite the value functions in terms of normalized productivity $\tilde{p} = p - gt$.

$$
\begin{aligned}
V_t(r, h_t, \tilde{p}) = \max \Bigg\{ & V_t(0, h_t, \tilde{p}_{\min}), \\
& r + \tilde{p} + h_t + gt + \frac{\delta(1-\kappa)}{1+\rho} V_{0,t+1}(h_t) \\
& + \frac{\delta\kappa}{1+\rho} \int_{\tilde{p}_{\min}}^{\tilde{p}_{\max}} E_t \left[(1-\beta)V_{0,t+1}(h_t) + \beta V_{t+1}(0, h_{t+1}, x)\right] dF(x) + \\
& + \frac{\lambda_1}{1+\rho} \int_{\tilde{p}-g}^{\tilde{p}_{\max}} E_t \left[(1-\beta)V_{t+1}(0, h_{t+1}, \tilde{p}-g) + \beta V_{t+1}(0, h_{t+1}, x)\right] dF(x) \\
& + \frac{\lambda_1}{1+\rho} \int_{q(r,h_t,\tilde{p})}^{\tilde{p}-g} E_t \left[(1-\beta)V_{t+1}(0, h_{t+1}, x) + \beta V_{t+1}(0, h_{t+1}, \tilde{p}-g)\right] dF(x) \\
& + \frac{1}{1+\rho} \left[1 - \mu - \delta - \lambda_1 \bar{F}(q(r, h_t, \tilde{p}))\right] E_t V_{t+1}(r, h_{t+1}, \tilde{p}-g) \Bigg\}
\end{aligned}
$$

and

$$
V_{0,t+1} = E_t V_{t+1}(0, h_{t+1}, \tilde{p}_{\min}).
$$

Normalize Value Functions We make the substitution $V_t(r, h_t, \tilde{p}) = \tilde{V}_t(r, h_t, \tilde{p}) + \frac{gt}{\rho} + \frac{g}{\rho^2}$ and observe that there is no dependence of $\tilde{V}_t$ on t, so we write $V_t(r, h_t, \tilde{p}) = \tilde{V}(r, h_t, \tilde{p}) + \frac{gt}{\rho} + \frac{g}{\rho^2}$.

The normalized value functions are

$$
\begin{aligned}
\tilde{V}(r, h_t, \tilde{p}) = \max \Bigg\{ & \tilde{V}(0, h_t, \tilde{p}_{\min}), \\
& r + \tilde{p} + h_t + \frac{\delta(1-\kappa)}{1+\rho} \tilde{V}_0(h_t) \\
& + \frac{\delta\kappa}{1+\rho} \int_{\tilde{p}_{\min}}^{\tilde{p}_{\max}} E_t \left[(1-\beta)\tilde{V}_0(h_t) + \beta \tilde{V}(0, h_{t+1}, x)\right] dF(x) + \\
& + \frac{\lambda_1}{1+\rho} \int_{\tilde{p}-g}^{\tilde{p}_{\max}} E_t \left[(1-\beta)\tilde{V}(0, h_{t+1}, \tilde{p}-g) + \beta \tilde{V}(0, h_{t+1}, x)\right] dF(x) \\
& + \frac{\lambda_1}{1+\rho} \int_{q(r,h_t,\tilde{p})}^{\tilde{p}-g} E_t \left[(1-\beta)\tilde{V}(0, h_{t+1}, x) + \beta \tilde{V}(0, h_{t+1}, \tilde{p}-g)\right] dF(x) \\
& + \frac{1}{1+\rho} \left[1 - \mu - \delta - \lambda_1 \bar{F}(q(r, h_t, \tilde{p}))\right] E_t \tilde{V}(r, h_{t+1}, \tilde{p}-g) \Bigg\}
\end{aligned}
$$

and

$$\tilde{V}_0 = E_t \tilde{V}(0, h_{t+1}, \tilde{p}_{\min}).$$

Solving for $\frac{\partial V(0,h_t,\tilde{p})}{\partial \tilde{p}}$ By integration by parts,

$$\begin{aligned}
\tilde{V}(r, h_t, \tilde{p}) = \max \Bigg\{ & \tilde{V}(0, h_t, \tilde{p}_{\min}), \\
& r + \tilde{p} + h_t + \frac{\delta}{1+\rho}\tilde{V}_0(h_t) \\
& + \frac{1}{1+\rho} E_t \Bigg\{ (1-\mu-\delta)\tilde{V}(r, h_{t+1}, \tilde{p} - g) \\
& + \lambda_1 \beta \int_{\tilde{p}-g}^{\tilde{p}_{\max}} \frac{\partial \tilde{V}}{\partial x}(0, h_{t+1}, x)\bar{F}(x)dx \\
& + \lambda_1 (1-\beta) \int_{q(r,h_t,\tilde{p})}^{\tilde{p}-g} \frac{\partial \tilde{V}}{\partial x}(0, h_{t+1}, x)\bar{F}(x)dx + \\
& + \delta\kappa\beta \int_{\tilde{p}_{\min}}^{\tilde{p}_{\max}} \frac{\partial \tilde{V}}{\partial x}(0, h_{t+1}, x)\bar{F}(x)dx \Bigg\} \Bigg\}
\end{aligned} \tag{1}$$

Plugging in $r = 0$ and using that $q(0, h_t, \tilde{p}) = \tilde{p} - g$,

$$\begin{aligned}
\tilde{V}(0, h_t, \tilde{p}) = \max \Bigg\{ & \tilde{V}(0, h_t, \tilde{p}_{\min}), \\
& \tilde{p} + h_t + \frac{\delta(1-\kappa)}{1+\rho}\tilde{V}_0(h_t) \\
& + \frac{1}{1+\rho} E_t \Bigg\{ (1-\mu-\delta)\tilde{V}(0, h_{t+1}, \tilde{p} - g)) \\
& + \lambda_1 \beta \int_{\tilde{p}-g}^{\tilde{p}_{\max}} \frac{\partial \tilde{V}}{\partial x}(0, h_{t+1}, x)\bar{F}(x)dx \\
& + \delta\kappa\beta \int_{\tilde{p}_{\min}}^{\tilde{p}_{\max}} \frac{\partial \tilde{V}}{\partial x}(0, h_{t+1}, x)\bar{F}(x)dx \Bigg\} \Bigg\}
\end{aligned}$$

Differentiating with respect to $\tilde{p}$, we get

$$\frac{\partial \tilde{V}}{\partial \tilde{p}}(0, h_t, \tilde{p}) = 1 + \frac{1-\mu-\delta-\lambda_1\beta\bar{F}(\tilde{p}-g)}{1+\rho} E_t \frac{\partial \tilde{V}}{\partial \tilde{p}}(0, h_{t+1}, \tilde{p}-g)$$

when $\tilde{V}(0, h_t, \tilde{p}) > \tilde{V}(0, h_t, \tilde{p}_{\min})$, i.e., when $\tilde{p} > \tilde{p}_{\min}$.

If we assume that h_t does not affect the marginal value of $\tilde{p}$ through some sophisticated expectational mechanism affecting bargaining, we have a functional equation

$$\frac{\partial \tilde{V}}{\partial p}(0, h_t, \tilde{p}) = 1 + \frac{1 - \mu - \delta - \lambda_1 \beta \bar{F}(\tilde{p} - g)}{1 + \rho} \frac{\partial \tilde{V}}{\partial \tilde{p}}(0, h_{t+1}, \tilde{p} - g).$$

This functional equation has a unique solution in $L_0([\tilde{p}_{\min}, \tilde{p}_{\max}])$. Before we proceed to show this, we need to determine what values $\bar{F}$ and $\partial \tilde{V}/\partial \tilde{p}$ have for $\tilde{p} < \tilde{p}_{\min}$. By naturally extending F, $\bar{F} = 1$ for $\tilde{p} < \tilde{p}_{\min}$. The marginal value of productivity is zero when $\tilde{p} < \tilde{p}_{\min}$ since the worker will voluntarily enter unemployment regardless. Therefore, $\partial \tilde{V}/\partial \tilde{p} = 0$ for $\tilde{p} < \tilde{p}_{\min}$.

Lemma C.1. *The functional equation*

$$f(x) = 1 + \frac{1 - \mu - \delta - \lambda_1 \beta \bar{F}(x - g)}{1 + \rho} f(x - g) \tag{2}$$

has a unique solution in $L_0([\tilde{p}_{\min}, \tilde{p}_{\max}])$*, with the convention* $f(x) = 0$ *and* $\bar{F}(x) = 1$ *for* $x < \tilde{p}_{\min}$.

Proof. Consider the functional mapping T defined by

$$Tf(x) = 1 + \frac{1 - \mu - \delta - \lambda_1 \beta \bar{F}(x - g)}{1 + \rho} f(x - g).$$

It is a contraction mapping under the sup norm. For any f, h, we have

$$\begin{aligned} |Tf(x) - Th(x)| &= \left| \frac{1 - \mu - \delta - \lambda_1 \beta \bar{F}(x - g)}{1 + \rho} \right| |f(x - g) - h(x - g)| \\ < \frac{1 - \mu - \delta}{1 + \rho} |f(x - g) - h(x - g)| &\leqslant \frac{1 - \mu - \delta}{1 + \rho} \sup_{x \in [p_{\min}, p_{\max}]} |f(x) - h(x)| \end{aligned}$$

so $\sup_{x \in [\tilde{p}_{\min}, \tilde{p}_{\max}]} |Tf(x) - Th(x)| < \rho_T \sup_{x \in [\tilde{p}_{\min}, \tilde{p}_{\max}]} |f(x) - h(x)|$ with $\rho_T = \frac{1 - \mu - \delta}{1 + \rho}$. Note that when $x - g < \tilde{p}_{\min}$, f and h coincide by definition.

Since the mapping is a contraction mapping under the sup norm, Banach's fixed point theorem applies and there exists a unique solution to $Tf = f$ in L_0. That is, the functional equation has a unique solution. □

Note that the contraction mapping suggests a straight-forward way to compute $\partial \tilde{V}(0, h_t, \tilde{p})/\partial \tilde{p}$ by iteration. Note also that the solution need not be continuous, in fact it is not. For example, when the productivity increases so that the individual will voluntarily become unemployed two periods into the future rather than one period into the future, the marginal value of productivity jumps (roughly doubles). These small discontinuities are an artifact of the discretization of time which would disappear if the model were framed in continuous time.

Solving for the Value Function Plugging in the f that solves Equation 2 for $\partial\tilde{V}/\partial\tilde{p}(0, h_{t+1}, x)$ in the integral of Equation 1, we get

$$
\begin{aligned}
\tilde{V}(r, \tilde{p}, h_t) = \max \Bigg\{ & \tilde{V}(0, \tilde{p}_{\min}, h_t), \\
& r + \tilde{p} + h_t + \frac{\delta}{1+\rho}\tilde{V}_0(h_t) \\
& + \frac{1}{1+\rho} E_t \Big\{ (1-\mu-\delta)\tilde{V}(r, \tilde{p}-g, h_{t+1}) \\
& + \lambda_1 \beta \int_{\tilde{p}-g}^{p_{\max}} f(x)\bar{F}(x)dx \\
& + \lambda_1 (1-\beta) \int_{q(r,\tilde{p},h_t)}^{\tilde{p}-g} f(x)\bar{F}(x)dx + \\
& + \delta\kappa\beta \int_{p_{\min}}^{p_{\max}} f(x)\bar{F}(x)dx \Big\} \Bigg\}
\end{aligned}
$$

with $\tilde{V}_0 == E_V(0, h_{t+1}, \tilde{p}_{\min})$.

Finally we show that there is a solution where decision functions do not depend on h_t. We expand the expression for $\tilde{V}(0, \tilde{p}_{\min}, h_t)$, and guessing that $\tilde{V}(r, \tilde{p}, h_t)$ can be separated into $\tilde{V}(r, \tilde{p}, h_t) = \tilde{V}(r, \tilde{p}) + \tilde{V}_2(h_t)$ (with some abuse of notation). Furthermore, assume that q only depends on r and $\tilde{p}$.

We then arrive at the equation

$$
\begin{aligned}
\tilde{V}(r, \tilde{p}) = \max \Bigg\{ & \tilde{V}(0, \tilde{p}_{\min}), \\
& r + \tilde{p} + h_t + \frac{\delta}{1+\rho}\tilde{V}_0 \\
& + \frac{1}{1+\rho} E_t \Big\{ (1-\mu-\delta)\tilde{V}(r, \tilde{p}-g) \\
& + \lambda_1 \beta \int_{\tilde{p}-g}^{p_{\max}} f(x)\bar{F}(x)dx \\
& + \lambda_1 (1-\beta) \int_{q(r,\tilde{p},h_t)}^{\tilde{p}-g} f(x)\bar{F}(x)dx + \\
& + \delta\kappa\beta \int_{p_{\min}}^{p_{\max}} f(x)\bar{F}(x)dx \Big\} \Bigg\}
\end{aligned}
\tag{3}
$$

which determine the policy functions q and the binary maximization choice.

Solving for the Value Function Numerically For a given q, Equation 3 can quickly be iterated (with the integrals pre-computed) to get a candidate $\tilde{V}$. We therefore iterate the expression until we have a converged candidate $\tilde{V}$, then compute a new policy function q, and iterate over until convergence. The algorithm is similar in spirit to Howard policy iteration.

Solving for Transitions We solve for the transition dynamics by standard value function iteration. To minimize the numerical discrepancy between our method for solving for the steady state and our value function iteration for the transition, we compute the integrals for the value function iteration using integration by parts as well.